Equation Summary
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Chemical Engineering 3453 Heat Transfer
University of Utah

Geometry

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\pi r = \pi D$</td>
<td>Circumference of circle</td>
</tr>
<tr>
<td>$4\pi r^2 = \pi D^2$</td>
<td>Area of circle</td>
</tr>
<tr>
<td>$4\pi r^2 = \pi D^2$</td>
<td>Volume of sphere</td>
</tr>
<tr>
<td>$\frac{\text{base}(\text{height})}{2}$</td>
<td>Area of triangle</td>
</tr>
</tbody>
</table>

Steady-State Conduction

Planar System

\[
\begin{align*}
\dot{Q} &= \frac{T_1 - T_2}{\frac{\Delta x_A}{k_A A} + \frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A}} \\
&= \frac{T_1 - T_2}{R_A + R_B + R_C}
\end{align*}
\]

Hollow Cylindrical System with Convection

\[
q = \frac{T_i - T_o}{\frac{1}{2\pi r_1 L h_i} + \frac{\ln(r_2 / r_1)}{2\pi k L} + \frac{1}{2\pi r_2 L h_o}} = \frac{T_i - T_o}{R_i + R_i + R_o} = \frac{T_i - T_2}{R_i}
\]
Spherical System

\[ q = \frac{T_1 - T_2}{(r_2 - r_1) / (4\pi kr_1r_2)} \]

Pin Fin with Insulated Tip

\[ kA \frac{d^2T}{dx^2} - hP(T - T_\infty) = 0; \quad T|_{x=0} = T_b; \quad \left. \frac{dT}{dx} \right|_{x=L} = 0 \]

\[ \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}, \quad \text{where } m = \left(\frac{hP}{kA}\right)^{1/2} \]

\[ q = kA_m(T_b - T_\infty) \tanh mL \]

\[ \eta_f = \frac{\tanh mL}{mL} \]

Conduction with Heat Source

\[ \frac{d}{dr} \left( r^{b-1} \frac{dT}{dr} \right) + \frac{\dot{q}}{k} r^{b-1} = 0; \quad \left. \frac{dT}{dr} \right|_{r=0} = 0; \quad -k \frac{dT}{dr} = h[T(R) - T_\infty] \]

\[ \frac{T(r) - T_\infty}{\dot{q} R^2 / k} = \frac{1}{2b} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] + \frac{1}{bBi} \quad \{ b = 1, \text{ plate, thickness } 2R \}
\quad \{ b = 2, \text{ cylinder, diameter } 2R \}
\quad \{ b = 3, \text{ sphere, diameter } 2R \} \]

where \( Bi = hR/k. \)

Transient Conduction

Governing Equations

Note that there is not a source term in the energy balance equation, the PDE.

\[ \frac{\partial T}{\partial t} = \frac{\alpha}{r^{b-1}} \frac{\partial}{\partial r} \left( r^{b-1} \frac{\partial T}{\partial r} \right) \quad \{ b = 1, \text{ plate, thickness } 2R \}
\quad \{ b = 2, \text{ cylinder, diameter } 2R \}
\quad \{ b = 3, \text{ sphere, diameter } 2R \} \]

for \( t < 0, \quad T = T_i \)

at \( r = 0, \quad \frac{\partial T}{\partial r} = 0 \)

at \( r = R, \quad -k \frac{\partial T}{\partial r} = h(T - T_\infty) \)
**Dimensionless Variables**

(1) temperature \( \theta / \theta_i = [T(r,t) - T_\infty] / (T_i - T_\infty) \); (2) heat loss fraction \( Q / Q_i = Q / [\rho c V(T_i - T_\infty)] \), where \( V \) is volume; (3) distance from center \( r^* = r / R \); (4) time \( Fo = \alpha t / R^2 \); (5) Biot number \( Bi = hR / k \), and (6) \( \zeta = \beta_n R \).

**Lumped Analysis (Bi < 0.1)**

\[
\frac{\theta}{\theta_i} = \exp \left( -\frac{t}{\tau} \right) \quad \text{and} \quad \frac{Q}{Q_i} = 1 - \exp \left( -\frac{t}{\tau} \right) \quad \text{and} \quad \tau = \frac{\rho c V}{hA} \quad \text{(characteristic time)}
\]

**Single-Term Solutions (Bi > 0.1 and Fo > 0.2)**

\[
\frac{\theta}{\theta_i} = C_1 \exp \left( -\zeta_1^2 Fo \right) S_1 \left( \zeta_1 r^* \right) \quad \text{and} \quad \frac{Q}{Q_0} (C_1 \text{ and } \zeta_1 \text{ from Table 5.1 of text})
\]

<table>
<thead>
<tr>
<th>Geometry</th>
<th>( S_1 )</th>
<th>( Q/Q_0 )</th>
<th>( \theta_0^* ) (centerline)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>( \cos \left( \zeta_1 r^* \right) )</td>
<td>1 - ( \sin \left( \zeta_1 \right) \theta_0^* / \zeta_1 )</td>
<td>( \theta_0^* = C_1 \exp \left( -\zeta_1^2 Fo \right) )</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( J_0 \left( \zeta_1 r^* \right) )</td>
<td>1 - ( 2\theta_0^* / \zeta_1 J_1 \left( \zeta_1 \right) )</td>
<td>( \theta_0^* = C_1 \exp \left( -\zeta_1^2 Fo \right) )</td>
</tr>
<tr>
<td>Sphere</td>
<td>( \sin \left( \zeta_1 r^* \right) / \zeta_1 r^* )</td>
<td>1 - ( 3\theta_0^* / \zeta_1^3 \left[ \sin \left( \zeta_1 \right) - \zeta_1 \cos \left( \zeta_1 \right) \right] )</td>
<td>( \theta_0^* = C_1 \exp \left( -\zeta_1^2 Fo \right) )</td>
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**The Semi-Infinite Solid**

Constant surface temperature: \( T(0,t) = T_s \)

\[
\frac{T(x,t) - T_s}{T_i - T_s} = \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) \quad \text{and} \quad q_s^* = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}
\]

Constant surface heat flux

\[
T(x,t) - T_i = \frac{2q''(\alpha t / \pi)^{1/2}}{k} \exp \left( -\frac{x^2}{4\alpha t} \right) - \frac{q''x}{k} \text{erfc} \left( \frac{x}{2\sqrt{\alpha t}} \right)
\]

Surface convection: \( k \frac{\partial T}{\partial x} \bigg|_{x=0} = h \left[ T(0,t) - T_\infty \right] \)

\[
\frac{T(x,t) - T_\infty}{T_i - T_\infty} = \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) + \exp \left( \frac{hx}{k} + \frac{h^2 \alpha t}{k^2} \right) \text{erfc} \left( \frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k} \right)
\]
<table>
<thead>
<tr>
<th>Bi</th>
<th>Plane Wall</th>
<th>Infinite Cylinder</th>
<th>Sphere</th>
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<tbody>
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<td>(C_1)</td>
<td>(\zeta_1) (rad)</td>
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</table>

*Bi = hL/k for the plane wall and hr/k for the infinite cylinder and sphere. See Figure 5.6.*