Review of Engineering Thermodynamics

Universal Balance Equation for Any Extensive Property

Accumulation = transport + generation

Integrated form for some period of time:

\[
\begin{bmatrix}
\text{final amount} \\
\text{initial amount}
\end{bmatrix} = \begin{bmatrix}
\text{amount entering} \\
\text{amount leaving} \\
\text{amount generated} \\
\text{amount consumed}
\end{bmatrix}
\]

Rate form:

\[
\begin{bmatrix}
\text{rate of change}
\end{bmatrix} = \begin{bmatrix}
\text{rate of transport in} \\
\text{rate of transport out} \\
\text{rate of generation} \\
\text{rate of consumption}
\end{bmatrix}
\]

Mass Balance

- Unsteady balance for CV

\[
\frac{dm_{CV}}{dt} = \sum_{\text{inlets}} m_i - \sum_{\text{exits}} m_e \\
\Delta m_{CV} = m_2 - m_1 = \sum_{\text{inlets}} m_i - \sum_{\text{exits}} m_e
\]

- Steady balance for CV

\[
0 = \sum_{\text{inlets}} m_i - \sum_{\text{exits}} m_e
\]

- Balance for closed system

\[
\frac{dm_{sys}}{dt} = 0 \\
\Delta m_{sys} = m_2 - m_1 = 0
\]

- Averaged flow

\[
\dot{m} = \rho_{av} Vel_{av} A = \frac{Vel_{av} A}{V_{av}} = \frac{V}{V_{av}}
\]

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Energy Balance

\[ e = u + \frac{1}{2} V e l^2 + g z, \quad h = u + P v \]

- Unsteady balance for CV

\[
\frac{dE_{CV}}{dt} = Q_{in,net} + \dot{W}_{in,net} + \sum_{\text{inlets}} \dot{m}_i \left( h + \frac{V e l^2}{2} + g z \right)_i - \sum_{\text{exits}} \dot{m}_e \left( h + \frac{V e l^2}{2} + g z \right)_e
\]

- Steady balance for CV

\[
0 = \dot{Q}_{in,net} + \dot{W}_{in,net} + \sum_{\text{inlets}} \dot{m}_i \left( h + \frac{V e l^2}{2} + g z \right)_i - \sum_{\text{exits}} \dot{m}_e \left( h + \frac{V e l^2}{2} + g z \right)_e
\]

- Balance for closed system

\[
\frac{dE_{sys}}{dt} = \dot{Q}_{in,net} + \dot{W}_{in,net} \quad \Delta \frac{E_{sys}}{sys} = Q_{in,net} + W_{in,net}
\]

Entropy Balance

There is only one form of entropy – internal entropy.

- Unsteady balance for CV

\[
dS_{CV} = \sum_{j=1}^{n} \frac{Q_{h,j}}{T_j} + \sum_{\text{inlets}} \dot{m}_i s_i - \sum_{\text{exits}} \dot{m}_e s_e + S_{gen} \]

\[ S_{gen} > 0 \text{ irreversible process} \]

\[ S_{gen} = 0 \text{ reversible process} \]

\[ S_{gen} < 0 \text{ impossible process} \]

- Steady balance for CV

\[
0 = \sum_{j=1}^{n} \frac{Q_{h,j}}{T_j} + \sum_{\text{inlets}} \dot{m}_i s_i - \sum_{\text{exits}} \dot{m}_e s_e + S_{gen}
\]

- Balance for closed system

\[
\frac{dS_{sys}}{dt} = \sum_{j=1}^{n} \frac{Q_{h,j}}{T_j} + S_{gen} \quad \Delta S_{sys} = m(s_2 - s_1) = \frac{1}{2} \frac{\partial Q_m}{T} + S_{gen}
\]

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Combined Entropy and Energy Balance

\[ du = \delta q + \delta w \quad ds = \frac{\delta q}{T} + ds_{\text{gen}} \]

\[ du = \delta q_{\text{rev}} + \delta w_{\text{rev}} \quad ds = \frac{\delta q_{\text{rev}}}{T} \quad \delta w_{\text{rev}} = -Pdv \]

\[ Tds = du + Pdv \quad \text{or} \quad ds = \frac{du}{T} + \frac{Pdv}{T} \]

An alternate form follows from the relation

\[ d(Pv) = Pdv + vdP \]

\[ Tds = dh - vdP \quad \text{or} \quad ds = \frac{dh}{T} - \frac{vdP}{T} \]

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Combined Entropy and Energy Balance for Ideal Gases

\[ \Delta s = \int_T^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_T}{v_1} \]

\[ \Delta s = \int_T^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_T}{P_1} \]

For constant or averaged heat capacities,

\[ \Delta s = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \]

\[ \Delta s = c_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \]

where \( T_{av} = \frac{T_1 + T_2}{2} \) and \( c_{av} = c(T_{av}) \)

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Equations for Work

Reversible boundary work, closed system:

\[ w_{\text{rev.in}} = -\int_{v_1}^{v_2} Pdv \]

Steady-flow, reversible work, open system:

\[ w_{\text{rev.in}} = \frac{W}{m} = \int_{P_1}^{P_2} vdp + \frac{V_{f}^{2} - V_{i}^{2}}{2} + g(z_{2} - z_{1}) \]

Models of Working Substances

<table>
<thead>
<tr>
<th>Solid/Liquid</th>
<th>Gases</th>
<th>Phase-change fluids (water &amp; refrigerants)</th>
<th>Mixtures (advanced)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p = C_v = C )</td>
<td>( \Delta u = \Delta h = C\Delta T )</td>
<td>( x = \frac{m_{vap}}{m_{tot}} = \frac{V - V_f}{V_f - V_i} )</td>
<td></td>
</tr>
</tbody>
</table>

**Perfect gas**
\( P V = RT \)
- \( P \) and \( T \) are absolute.
- \( C_p = \text{constant} \)
- \( C_v = C_p - R \)
- \( \Delta u = C_v \Delta T \)
- \( \Delta h = C_p \Delta T \)

**Ideal gas**
\( P V = RT \)
- \( P \) and \( T \) are absolute.
- \( C_p = f(T) \)
- \( C_v = C_p - R \)
- \( \Delta u = \int_{T_i}^{T_f} C_v dT \)
- \( \Delta h = \int_{V_i}^{V_f} C_p dV \)

**Real gas**
\( Z = \frac{f(T, P)}{v_{\text{actual}}/v_{\text{ideal}}} \)
- \( Z = \frac{P v_{\text{actual}}}{RT} \)
- \( T_R = \frac{T}{T_c} \)
- \( P_R = \frac{P}{P_c} \)
- \( P \) and \( T \) are absolute.


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