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H. The Tds Relations and Changes in Entropy

4. Example

Two pieces of copper, A and B, with masses 1 and 3 kg, and initial temperatures 0 and 200°C are brought together and allowed to equilibrate while insulated from the surroundings. Determine (a) the entropy change of A and B and (b) the entropy generation ($S_{gen}$) for the process, in kJ/K.

Entropy change given by (7-28):

$$\Delta S = mC_v \ln \frac{T_2}{T_1}$$

Final temperature, $T_2$, is obtained from energy balance.

$$\Delta U = Q - W = 0$$
$$[mC_v(T_2 - T_1)]_A + [mC_v(T_2 - T_1)]_B = 0$$
$$T_1(T_2 - 0) + 3(T_2 - 200) = 0$$
$$T_2 = 150 \text{ C} \quad (151 \text{ C if use } C_v \text{ for each})$$

Entropy changes for A and B are

$$\Delta S_A = 1(0.390) \frac{kJ}{kg} \ln \frac{423}{273} = 0.171 \frac{kJ}{K}$$

$$\Delta S_B = 3(0.400) \frac{kJ}{kg} \ln \frac{423}{473} = -0.134 \frac{kJ}{K}$$
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4. Example

Entropy generation from overall entropy balance on composite system (7-9)

\[(S_2 - S_1)_{sys} = \int \frac{\delta Q}{T} + S_{gen}\]

There is no heat transfer (\(\delta Q = 0\)) so

\[S_{gen} = (S_2 - S_1)_{sys} = \Delta S_A + \Delta S_B = 0.171 + (-0.134) = 0.037 \text{kJ/K}\]

Because \(S_{gen} > 0\), the process is irreversible.

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5. Isentropic processes with ideal gases
   a. approximate treatment assuming constant specific heats
      i. first isentropic relation (applies to open and closed systems), recall

\[\Delta s = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}\]

(ideal gas, 7-33)

Reversible and adiabatic \(\Rightarrow\) isentropic \(\Rightarrow\) \(\Delta s = 0\).

(7-33) becomes (remember \(R = c_p - c_v\), \(k = c_p/c_v\))

\[c_{v,av} \ln \frac{T_2}{T_1} = R \ln \frac{V_1}{V_2}\]

or

\[\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{c_v}} = \left(\frac{V_1}{V_2}\right)^{k-1}\]

(ideal gas, isentropic, 7-42)

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ii. second isentropic relation (applies to open and closed systems)

\[ \Delta s = c_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \]  
(ideal gas, 7-34)

Reversible and adiabatic \( \Rightarrow \) isentropic \( \Rightarrow \Delta s = 0 \).

(6.23) becomes (remember \( R = c_p - c_v \, k = c_p/c_v \))

\[ c_{p,av} \ln \frac{T_2}{T_1} = R \ln \frac{P_2}{P_1} \quad \text{or} \quad \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{R/c_p} = \left( \frac{P_2}{P_1} ^{\frac{k}{k-1}} \right) \]

(ideal gas, isentropic, 7-43)

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iii. third isentropic relation (applies to open and closed systems)

From (7-42) and (7-43),

\[ \frac{P_2}{P_1} ^{\frac{R}{c_p}} = \left( \frac{v_1}{v_2} \right)^k \]  
(ideal gas, isentropic, 7-44)

Equations 7-42 – 7-44 can be written as

\[ Tv^{k-1} = C \]
\[ TP^{\frac{(k-1)}{k}} = C \]
\[ Pv^k = C \]

(ideal gas, isentropic, where \( C \) is a constant.)

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b. exact treatment using variable specific heats

\[ s_2 - s_1 = s^o_2 - s^o_1 - R \ln \frac{P_2}{P_1} \]  

(ideal gas, 7-39)

Reversible and adiabatic \( \Rightarrow \) isentropic \( \Rightarrow \Delta s = 0 \).

(7-39) becomes

\[ \frac{P_2}{P_1} = \exp \left( \frac{s^o_2 - s^o_1}{R} \right) = \exp \left( \frac{s^0_2}{R} \right) = \frac{P_{2,s}}{P_{1,s}} \]

(ideal gas, isentropic, 7-49)

where \( P_i \) is called the relative pressure (Table A-17).

We can also define \( v_r \), the relative specific volume (Table A-17).

\[
\begin{pmatrix}
  v_2 \\
  v_1
\end{pmatrix}
\]

s-constr.

\[
\begin{pmatrix}
  v_{12} \\
  v_{r1}
\end{pmatrix}
\]

(ideal gas, isentropic, 7-50)

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Reversible and Actual Work and the Generation of Entropy

Consider two, steady flow processes, one reversible and the other irreversible (actual), with the same inlet and exit conditions.

\[ h_1, s_1 \quad \rightarrow \quad 1 \quad \rightarrow \quad 2 \quad \rightarrow \quad h_2, s_2 \]

Energy balances:
\[ \dot{m}(h_1 - h_2) = \dot{W}_{\text{out,act}} - \dot{Q}_{\text{in,act}} = \dot{W}_{\text{out,rev}} - \dot{Q}_{\text{in,rev}} \]

Entropy balances:
\[ \dot{Q}_{\text{in,act}} = \dot{m}T(s_2 - s_1) - TS_{\text{gen}} \quad \text{and} \quad \dot{Q}_{\text{in,rev}} = \dot{m}T(s_2 - s_1) \]

Conclusion:
\[ \therefore W_{\text{out,rev}} = W_{\text{out,act}} + TS_{\text{gen}} \]

\(TS_{\text{gen}}\) represents the rate at which work is "lost" due to irreversibilities.
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K. Reversible, Steady-Flow Work

1. Steady-flow balance equations (applied to a compressor)

\[
0 = \dot{m}_1 - \dot{m}_2
\]

\[
\dot{Q}_{\text{cv, in}} + W_{\text{nonflow, in}} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]
\]

\[
\dot{m}_1 s_1 - \dot{m}_2 s_2 + \frac{\dot{Q}}{T} + \dot{S}_\text{gen} = 0
\]

Divide the energy and entropy balances by \( \dot{m} \):

\[
q + w_{\text{in}} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)
\]

\[
s_1 - s_2 + \frac{q_{\text{rev}}}{T} = 0
\]

where \( \dot{Q} = \dot{m}q, \dot{W} = \dot{m}w, \text{and} \ \dot{S}_\text{gen} = 0 \).

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In differential form

\[
\delta q + \delta w_{\text{in}} = dh + d\left(\frac{V^2}{2}\right) + gdz \quad \text{and} \quad -ds + \frac{\delta q_{\text{rev}}}{T} = 0
\]

Rearrange the later to give \( \delta q_{\text{rev}} = Tds \)

Recall that \( Tds = dh - vdP \) \((7\text{-24})\)

Then \( \delta w_{\text{rev, in}} = vdP + d(ke) + d(pe) \)
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Integrating from 1 to 2 gives

$$w_{rev,in} = \int_1^2 v dP + \Delta ke + \Delta pe$$  \hspace{1cm} (7-51)

For a compressor, $w_{rev,in}$ is a positive number.
For a turbine, $w_{rev,in}$ is a negative number.
Neglecting $\Delta ke$ and $\Delta pe$ gives

$$w_{rev,in} = \int_1^2 v dP$$  \hspace{1cm} (7-52)

For a turbine, it is convenient to rewrite (7-51) as

$$w_{rev,out} = -\int_1^2 v dP - \Delta ke - \Delta pe$$