VII. Power and Refrigeration Cycles

C. The Carnot Power Cycle for an Ideal Gas

1. Description

- 1-2 reversible, isothermal expansion
- 2-3 reversible, adiabatic expansion
- 3-4 reversible, isothermal compression
- 4-1 reversible, adiabatic compression

\[ T_H = \text{constant} \]
\[ T_L = \text{constant} \]

\[ Q_H \]
\[ Q_L \]

\[ W_{\text{net,out}} = 0 \]

\[ Q_{23} = 0 \]

\[ s_2 = s_3 \]

\[ s_4 = s_1 \]

2. Analysis

- 1-2 rev., isothermal expansion
  \[ Q_H = -W_{1-2} = mRT_H \ln \frac{v_2}{v_1} \]
- 2-3 rev., adiabatic expansion
  \[ Q_{2-3} = 0 \]
- 3-4 rev., isothermal compression
  \[ Q_L = W_{3-4} = mRT_L \ln \frac{v_3}{v_4} \]
- 4-1 rev., adiabatic compression
  \[ Q_{4-1} = 0 \]

\[ \eta_{\text{th,rev}} = \frac{W_{\text{net,out}}}{Q_H} = 1 - \frac{Q_L}{Q_H} \]

\[ \eta_{\text{th,rev}} = 1 - \frac{mRT_L \ln \frac{v_3}{v_4}}{mRT_H \ln \frac{v_2}{v_1}} \]
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2. Analysis (cont.)

\[ \frac{T_3}{T_2} = \frac{T_L}{T_H} \left( \frac{v_2}{v_3} \right)^{k-1} \]

\[ \frac{T_4}{T_1} = \frac{T_L}{T_H} \left( \frac{v_1}{v_4} \right)^{k-1} \]

\[ \frac{v_3}{v_4} = \frac{v_2}{v_1} \]

This was derived for an ideal gas but applies to all working substances.

\[ \eta_{th,rev} = 1 - \frac{mRT_L \ln \frac{v_2}{v_4}}{mRT_H \ln \frac{v_2}{v_1}} = 1 - \frac{T_L}{T_H} \]  

(6-18)

D. Gas Power Cycle for Spark Ignition, Internal Combustion Engines (Otto Cycle)

1. Definitions
   a. top dead center (TDC)
   b. bottom dead center (BDC)
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1. Definitions (cont.)
   c. mean effective pressure
   \[ W_{\text{net, out}} = (\text{mep})(V_{\max} - V_{\min}) \]
   d. compression ratio
   \[ r = \frac{V_{\max}}{V_{\min}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}} \]

2. The Air Standard Otto Cycle. The Otto cycle is used to model two- and four-stroke engines. The working fluid is air.

   1-2 Isentropic compression. Flywheel carries piston into cylinder to give \( w_{\text{in}} \).

   2-3 Isometric heat addition. Combustion of gasoline provides heat addition \( q_{\text{in}} \).

   3-4 Isentropic expansion. Hot gas expands against piston to do work \( w_{\text{out}} \).

   4-1 Isometric heat removal. In four-stroke engines, hot gases are exhausted (1-0) and fresh air is drawn in (0-1). Steps (1-0) and (0-1) are not part of the two-stroke Otto cycle.
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3. Efficiency of Otto Cycle (Two- and Four-Stroke)
   a. Otto cycle is closed
   
   \[ \eta_{\text{th,otto}} = \frac{w_{\text{net.out}}}{q_{\text{in}}} = \frac{q_{\text{in}} - q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} \]
   
   b. heat transfer occurs at constant volume
   
   \[ q_{\text{in}} = u_3 - u_2 = C_v (T_3 - T_2) \]
   \[ q_{\text{out}} = u_4 - u_1 = C_v (T_4 - T_1) \]
   
   \[ \therefore \eta_{\text{th,otto}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} \left( \frac{T_4}{T_3} - 1 \right) \]

   c. power stroke (3-4) and compression stroke (1-2) are isentropic with

   \[ v_2 = v_3 \quad \text{and} \quad \eta_{\text{th,otto}} = 1 - \frac{T_1}{T_2} \left( \frac{T_4}{T_3} - 1 \right) \]

   Then

   \[ \frac{T_1}{T_2} = \left( \frac{v_2}{v_1} \right)^{k-1} = \left( \frac{v_3}{v_4} \right)^{k-1} = \frac{T_4}{T_3} \]

   \[ \eta_{\text{th,otto}} = 1 - \frac{T_1}{T_2} \left( \frac{T_4}{T_3} - 1 \right) = 1 - \frac{1}{r^{k-1}} \]

   where \( k = \frac{C_p}{C_v} \) and \( r = \frac{v_1}{v_2} \)
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d. conclusions for Otto cycle

- $\eta$ increases with increasing compression ratio, $r$, and $k$
- typical values of $r$ are 7 to 10
- for $r = 8$ and $k = 1.4$, $\eta_{\text{th,otto}} = 56.5\%$
- actual efficiencies are 25 to 30%