# Symbolic Calculus

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
</table>
| `diff(f,x)` | Take the first derivative of \( f \) with respect to \( x \). | \[
\text{syms } a \ x; \\
y = \sin(a*x); \\
diff(y,x); \\
diff(\cos(a*x), x )
\] | \[
\frac{d}{dx} (\sin(ax)) = a \cos(ax) \\
\frac{d}{dx} (\cos(ax)) = -a \sin(ax)
\] |
| `diff(f,x,n)` | Take the \( n \)th derivative of \( f \) with respect to \( x \). | \[
\text{syms } a \ x; \\
diff( \exp(-a*x), x, 3 )
\] | \[
\frac{d^3}{dx^3} e^{-ax} = -a^3 e^{-ax}
\] |
| `int(f,x)` | Indefinite integral of \( f \) with respect to \( x \). | \[
\text{syms } x; \\
\text{int}( 1/x, x )
\] | \[
\int \frac{dx}{x} = \ln x + C
\] |
| `int(f,x,lo,hi)` | Definite integral of \( f \) with respect to \( x \) from \( lo \) to \( hi \). | \[
\text{syms } a \ b \ c \ x; \\
\text{int}( \cos(c*x), x, a, b )
\] | \[
\int_a^b \cos(c \ x) \, dx = \frac{1}{c} [\sin(c \ b) - \sin(c \ a)]
\] |
Differential Equations

ODEs often arise in engineering applications:
✓ Kinetics (reaction engineering)
✓ Heat transfer
✓ Mass transfer
✓ Fluid mechanics
✓ Process control

Example: kinetics

\[
A \xrightarrow{k} B \quad \text{First order reaction}
\]

\[
\frac{dc_A}{dt} = -\frac{dc_B}{dt} = -kc_A
\]

Analytic solution...

2\(A \xrightarrow{k} B\) Second order reaction

\[
\frac{dc_A}{dt} = -\frac{dc_B}{dt} = -kc_A^2
\]

Analytic solution...

Example: Newton’s Law

\[F = ma\]

\[v \equiv \frac{dy}{dt} \quad a \equiv \frac{dv}{dt}\]

\[m\frac{dv}{dt} = \sum F \quad m\frac{d^2y}{dt^2} = \sum F\]

Analytic solution for constant \(a\)...
Symbolic Solution of Differential Equations

**Single ODE:**  \( \frac{dy}{dt} = f(y, t) \)

- \( y = \text{dsolve}('Dy=f') \)
- \( \text{dsolve}( 'Dy=f', 'y(0)=y_0' ) \)
- Solves the ODE and returns the solution.
- Note special syntax “Dy” for \( \frac{dy}{dt} \).

**Two ODEs:**  \( \frac{dy_1}{dt} = f_1(y, t), \quad \frac{dy_2}{dt} = f_2(y, t) \)

- \([y_1, y_2] = \text{dsolve}('Dy1=f1', 'Dy2=f2')\)
- Solves the ODEs and returns the solution as a two-entry vector.

**Three (or more) ODEs:**  \( \frac{dy_1}{dt} = f_1(y, t), \quad \frac{dy_2}{dt} = f_2(y, t), \quad \frac{dy_3}{dt} = f_3(y, t) \)

- \( S = \text{dsolve}( 'Dy1=f1', 'Dy2=f2', 'Dy3=f3' ) \)
- Solves the system of ODEs and returns the solution in a STRUCTURE:
  - \( S.y1 \) - solution for \( y_1 \).
  - \( S.y2 \) - solution for \( y_2 \).
  - \( S.y3 \) - solution for \( y_3 \).

\[ \frac{dca}{dt} = -kca \]

\text{syms ca k;}
ca = \text{dsolve}( 'Dca=-k*ca' );
\text{pretty(ca);}
Integral Transforms

**Fourier Transform** (many applications, including solution of ODEs)

\[ F(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} \, dx \quad \text{“forward” transform} \quad F=\text{fourier}(f,x) \]

\[ f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w)e^{iwx} \, dw \quad \text{“reverse” transform} \quad f=\text{ifourier}(F,w) \]

**Laplace Transform** (often used in solution of ODEs)

\[ F(s) = \int_{0}^{\infty} f(t)e^{-ts} \, dt \quad \text{“forward” transform} \quad F=\text{laplace}(f,t) \]

\[ f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s)e^{ts} \, ds \quad \text{“reverse” transform} \quad f=\text{ilaplace}(f,s) \]
Vectorize

\texttt{vectorize(a)}

- Convert expression \(a\) into a string.
- Can be used to create a function!

**Example: quadratic polynomial**

```plaintext
syms a b c x;
y = a*x^2 + b*x + c;
s = solve(y,x);
fprintf('%s
', vectorize(s))
```

\[ y = ax^2 + bx + c \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

1. Rip out the beginning “\texttt{matrix(}” and the end “\texttt{)}”
2. Copy the remaining output into a function.
3. Add function arguments and name as necessary.
4. Save the new function.