

Symbolic Calculus

Command	Description	Example
diff(f,x)	Take the first derivative of f with respect to x .	<pre>syms a x; y = sin(a*x); diff(y,x); diff(cos(a*x), x)</pre>
diff(f,x,n)	Take the n^{th} derivative of f with respect to x .	<pre>syms a x; diff(exp(-a*x), x, 3)</pre>
int(f,x)	Indefinite integral of f with respect to x .	<pre>syms x; int(1/x, x)</pre>
int(f,x,lo,hi)	Definite integral of f with respect to x from lo to hi .	<pre>syms a b c x; int(cos(c*x), x, a, b)</pre>

$$\frac{d}{dx} (\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx} (\cos(ax)) = -a \sin(ax)$$

$$\frac{d^3}{dx^3} e^{-ax} = -a^3 e^{-ax}$$

$$\int \frac{dx}{x} = \ln x + C$$

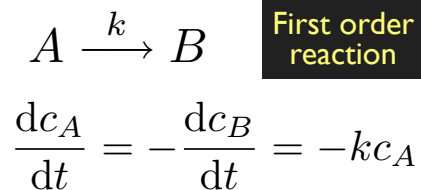
$$\int_a^b \cos(cx) dx = \frac{1}{c} [\sin(cb) - \sin(ca)]$$

Differential Equations

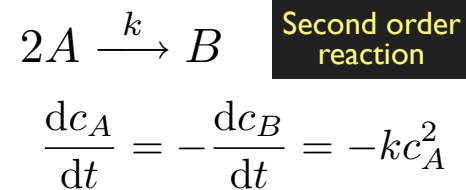
ODEs often arise in engineering applications:

- ✓ Kinetics (reaction engineering)
- ✓ Heat transfer
- ✓ Mass transfer
- ✓ Fluid mechanics
- ✓ Process control

Example: kinetics



Analytic solution...



Analytic solution...

Example: Newton's Law $F = ma$

$$v \equiv \frac{dy}{dt} \quad a \equiv \frac{dv}{dt} \quad \longrightarrow \quad m \frac{dv}{dt} = \sum F \quad m \frac{d^2y}{dt^2} = \sum F$$

Analytic solution for constant a ...

Symbolic Solution of Differential Equations

Single ODE: $\frac{dy}{dt} = f(y, t)$

- `y=dsolve('Dy=f')`
- `dsolve('Dy=f', 'y(0)=y0')`
- Solves the ODE and returns the solution.
- Note special syntax “Dy” for dy/dt .

$$\frac{dc_A}{dt} = -k c_A$$

```
syms ca k;  
ca=dsolve('Dca=-k*ca');  
pretty(ca);
```

Two ODEs: $\frac{dy_1}{dt} = f_1(y, t), \quad \frac{dy_2}{dt} = f_2(y, t)$

- `[y1,y2]=dsolve('Dy1=f1','Dy2=f2')`
- Solves the ODEs and returns the solution as a two-entry vector.

Three (or more) ODEs: $\frac{dy_1}{dt} = f_1(y, t), \quad \frac{dy_2}{dt} = f_2(y, t), \quad \frac{dy_3}{dt} = f_3(y, t)$

- `S=dsolve('Dy1=f1','Dy2=f2','Dy3=f3')`
- Solves the system of ODEs and returns the solution in a STRUCTURE:
 - `S.y1` - solution for y_1 .
 - `S.y2` - solution for y_2 .
 - `S.y3` - solution for y_3 .

Integral Transforms

Fourier Transform (many applications, including solution of ODEs)

$$F(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx \quad \text{"forward" transform} \quad F = \mathbf{fourier}(f, x)$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w)e^{iwx} dw \quad \text{"reverse" transform} \quad f = \mathbf{ifourier}(F, w)$$

Laplace Transform (often used in solution of ODEs)

$$F(s) = \int_0^{\infty} f(t)e^{-ts} dt \quad \text{"forward" transform} \quad F = \mathbf{laplace}(f, t)$$

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s)e^{ts} ds \quad \text{"reverse" transform} \quad f = \mathbf{ilaplace}(F, s)$$

Vectorize



vectorize(a)

- Convert expression a into a string.
- Can be used to create a function!

Example: quadratic polynomial

```
syms a b c x;  
y = a*x^2 + b*x + c;  
s = solve(y,x);  
fprintf('%s\n',vectorize(s))
```

$$y = ax^2 + bx + c \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```
matrix([[ -1./2.*(b-(b.^2-4.*c.*a).^(1./2))./a], [-1./2.*(b+(b.^2-4.*c.*a).^(1./2))./a]])
```

1. Rip out the beginning “**matrix(**” and the end “**)**”
2. Copy the remaining output into a function.
3. Add function arguments and name as necessary.
4. Save the new function.