Symbolic Calculus

Command	Description	Example	

`diff(f,x)`	Take the first derivative of **f** with respect to **x**.	`syms a x;`
		`y = sin(a*x);`
		`diff(y,x);`
		
		`diff( cos(a*x), x )`
			$\frac{\mathrm{d}}{\mathrm{d}x}(\sin(ax)) = a\cos(ax)$
			$\frac{\mathrm{d}}{\mathrm{d}x}(\cos(ax)) = -a\sin(ax)$

`diff(f,x,n)`	Take the n^{th} derivative of **f** with respect to **x**.	`syms a x;` `diff( exp(-a*x), x, 3 )`	$\frac{\mathrm{d}^3}{\mathrm{d}x^3}\mathrm{e}^{-ax} = -a^3\mathrm{e}^{-ax}$
`int(f,x)`	Indefinite integral of **f** with respect to **x**.	`syms x;` `int( 1/x, x )`	$\int \frac{\mathrm{d}x}{x} = \ln x + C$
`int(f,x,lo,hi)`	Definite integral of **f** with respect to **x** from **lo** to **hi**.	`syms a b c x;` `int( cos(c*x),x,a,b )`	$\int_a^b \cos(c\,x)\mathrm{d}x = \frac{1}{c}\left[\sin(c\,b) - \sin(c\,a)\right]$

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Differential Equations

ODEs often arise in engineering applications:

- ✓ Kinetics (reaction engineering)
- ✓ Heat transfer
- ✓ Mass transfer
- ✓ Fluid mechanics
- ✓ Process control

Example: kinetics

$$A \xrightarrow{k} B$$

First order reaction

$$\frac{\mathrm{d}c_A}{\mathrm{d}t} = -\frac{\mathrm{d}c_B}{\mathrm{d}t} = -kc_A$$

Analytic solution...

$$2A \xrightarrow{k} B$$

Second order reaction

$$\frac{\mathrm{d}c_A}{\mathrm{d}t} = -\frac{\mathrm{d}c_B}{\mathrm{d}t} = -kc_A^2$$

Analytic solution...

Example: Newton's Law $F = ma$

$$v \equiv \frac{\mathrm{d}y}{\mathrm{d}t} \qquad a \equiv \frac{\mathrm{d}v}{\mathrm{d}t} \quad \longrightarrow \quad m\frac{\mathrm{d}v}{\mathrm{d}t} = \sum F \qquad m\frac{\mathrm{d}^2y}{\mathrm{d}t} = \sum F$$

Analytic solution for constant a...

Symbolic Solution of Differential Equations

Single ODE: $\frac{\mathrm{d}y}{\mathrm{d}t} = f(y,t)$

- `y=dsolve('Dy=f')`
- `dsolve('Dy=f','y(0)=yo')`
- Solves the ODE and returns the solution.
- Note special syntax "`Dy`" for $\mathrm{d}y/\mathrm{d}t$.

$$\frac{\mathrm{d}c_A}{\mathrm{d}t} = -kc_A$$

```
syms ca k;
ca=dsolve('Dca=-k*ca');
pretty(ca);
```

Two ODEs: $\frac{\mathrm{d}y_1}{\mathrm{d}t} = f_1(y,t), \quad \frac{\mathrm{d}y_2}{\mathrm{d}t} = f_2(y,t)$

- `[y1,y2]=dsolve('Dy1=f1','Dy2=f2')`
- Solves the ODEs and returns the solution as a two-entry vector.

Three (or more) ODEs: $\frac{\mathrm{d}y_1}{\mathrm{d}t} = f_1(y,t), \quad \frac{\mathrm{d}y_2}{\mathrm{d}t} = f_2(y,t), \quad \frac{\mathrm{d}y_3}{\mathrm{d}t} = f_3(y,t)$

- `S=dsolve('Dy1=f1','Dy2=f2','Dy3=f3')`
- Solves the system of ODEs and returns the solution in a STRUCTURE:
  - `S.y1` - solution for `y1`.
  - `S.y2` - solution for `y2`.
  - `S.y3` - solution for `y3`.

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Integral Transforms

Fourier Transform (many applications, including solution of ODEs)

$$F(w) = \int_{-\infty}^{\infty} f(x) e^{-iwx} \mathrm{d}x$$ "forward" transform `F=fourier(f,x)`

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) e^{iwx} \mathrm{d}w$$ "reverse" transform `f=ifourier(F,w)`

Laplace Transform (often used in solution of ODEs)

$$F(s) = \int_{0}^{\infty} f(t) e^{-ts} \mathrm{d}t$$ "forward" transform `F=laplace(f,t)`

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s) e^{ts} \mathrm{d}s$$ "reverse" transform `f=ilaplace(f,s)`

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Vectorize

- `vectorize(a)`
  - Convert expression a into a string.
  - Can be used to create a function!

Example: quadratic polynomial

```
syms a b c x;
y = a*x^2 + b*x + c;
s = solve(y,x);
fprintf('%s\n',vectorize(s))
```

$$y = ax^2 + bx + c \;\Rightarrow\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```
matrix([[-1./2.*(b-(b.^2-4.*c.*a).^(1./2))./a],[-1./2.*(b+(b.^2-4.*c.*a).^(1./2))./a]])
```

1. Rip out the beginning "`matrix(`" and the end "`)`"
2. Copy the remaining output into a function.
3. Add function arguments and name as necessary.
4. Save the new function.