Models for Mass Transfer in "Wall" Bounded Turbulent Flows

CHEN 6603

References:

- S. B. Pope. Turbulent Flows. Cambridge University Press, New York, 2000.
- T&K §10.2

Overview

Model for molecular diffusion

MODEL for effects of turbulent mixing

$$
\begin{aligned}
\mathbf{j}_{i,\mathrm{net}} &= \mathbf{j}_i + \mathbf{j}_{i,\mathrm{turb}} \\
&= -\rho \sum_{j=1}^{n-1} \left(D^{\circ}_{ij} + \delta_{ij} D_{j,\mathrm{turb}} \right) \nabla \bar{\omega}_j \\
&\approx \rho \sum_{j=1}^{n-1} \left(k^{\bullet}_{ij} + \delta_{ij} k^{\bullet}_{j,\mathrm{turb}} \right) \Delta \bar{\omega}_j
\end{aligned}
$$

Mass-transfer coefficient approach - only consider net effects of mass transfer over some length.

- Turbulent boundary layers - why?
  - Often in chemical processes, we require mass transfer at interfaces
  - Turbulent flow at these interfaces ("walls") increases the rates of mass transfer by increasing gradients.
  - Close to walls, molecular viscosity & diffusivity dominate ($L/\eta \rightarrow 1$)
  - Away from walls (at large Re), "turbulent diffusivity" dominates.
- "Law" of the wall - describes length & time scales for turbulence near walls.
  - Objective: determine the turbulent viscosity (eddy viscosity) from physical arguments. Then use $\mathrm{Sc}_{\mathrm{turb}}$ to relate ν_{turb} and k_{turb} or D_{turb}.

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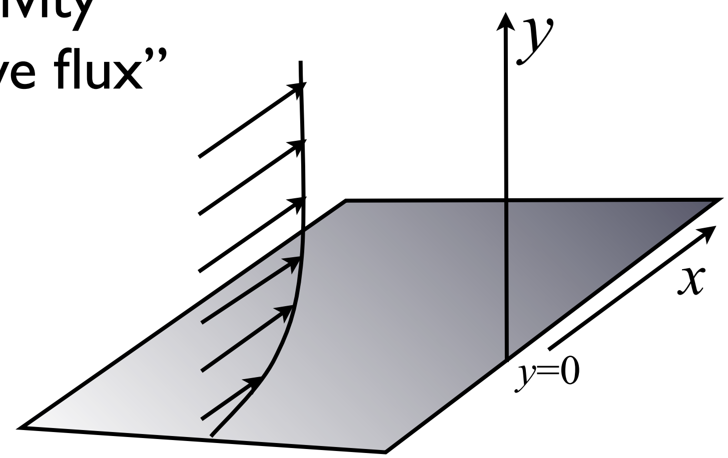
The quest for ν_{turb}.

<u>Key parameters</u> (near the wall):

- ν (kinematic viscosity) - “momentum diffusivity”
- τ_w (wall shear stress) - “momentum diffusive flux”

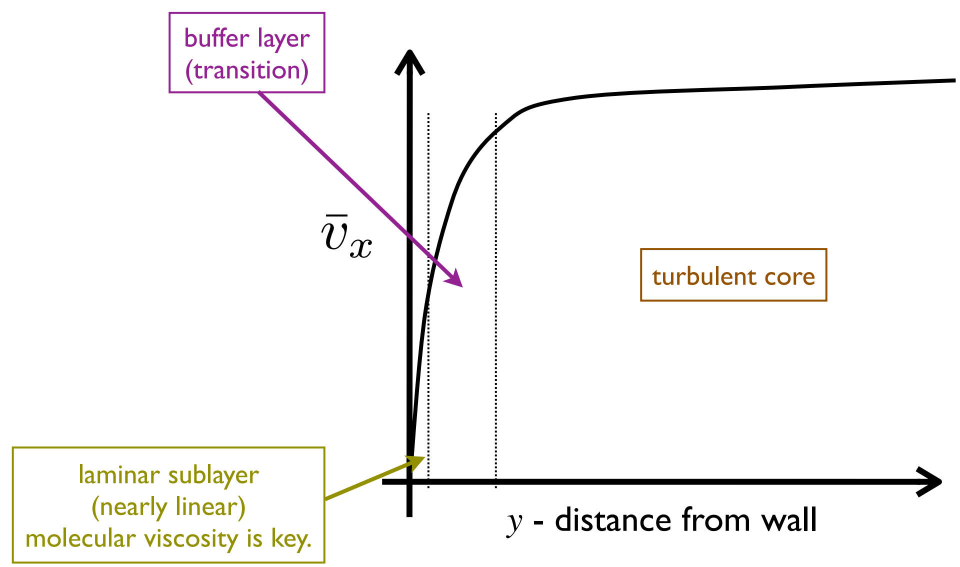


Time-averaged velocity profile.

Wall shear stress: $$\tau_w \equiv \rho\nu \left(\frac{\partial \bar{v}_x}{\partial y}\right)_{y=0}$$

Friction velocity: $$v_\tau \equiv \sqrt{\tau_w/\rho}$$

viscous length scale: $$\delta_\nu \equiv \nu\sqrt{\frac{\rho}{\tau_w}} = \frac{\nu}{v_\tau}$$

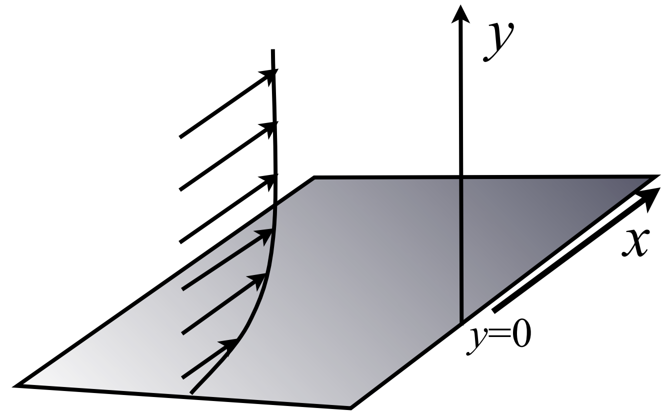


Distance from the wall in wall units (“viscous lengths”): $$y^+ \equiv \frac{y}{\delta_\nu} = \frac{v_\tau y}{\nu}$$

Velocity in wall units: $$v^+ \equiv \frac{\bar{v}_x}{v_\tau}$$

RANS Momentum equations, constant ρ, μ, no body forces:

$$\frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} - (\nu + \nu_t)\frac{\partial}{\partial x_i}\frac{\partial \bar{v}_j}{\partial x_j} = 0$$



Time-averaged velocity profile.

y, z directions have constant mean velocities = 0.

x-momentum equation is only relevant one.

$$\frac{\partial}{\partial x}(\bar{v}_x \bar{v}_x) + \frac{\partial}{\partial y}(\bar{v}_x \bar{v}_y) + \frac{\partial}{\partial z}(\bar{v}_x \bar{v}_z) + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} = (\nu + \nu_t)\left(\frac{\partial^2}{\partial x^2}\bar{v}_x + \frac{\partial^2}{\partial y^2}\bar{v}_x + \frac{\partial^2}{\partial z^2}\bar{v}_x\right)$$

$$\Downarrow$$

$$\frac{\partial}{\partial y}(\bar{v}_x \bar{v}_y) + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} = (\nu + \nu_t)\left(\frac{\partial^2}{\partial y^2}\bar{v}_x\right)$$

Near-wall shear stress

Implies this form for τ_w.

$$\frac{1}{\rho}\tau_w = (\nu + \nu_t)\frac{\partial \bar{v}_x}{\partial y}$$

$$= (\nu + \nu_t)\frac{\partial v_x^+}{\partial y^+}\frac{\partial \bar{v}_x}{\partial v_x^+}\frac{\partial y^+}{\partial y}$$

definition of y^+: $$y^+ \equiv \frac{y}{\delta_\nu} = \frac{v_\tau y}{\nu} \Rightarrow \frac{\partial y^+}{\partial y} = \frac{v_\tau}{\nu}$$

definition of v^+: $$v^+ \equiv \frac{\bar{v}_x}{v_\tau} \Rightarrow \frac{\partial \bar{v}_x}{\partial v_x^+} = v_\tau$$

$$\frac{\partial y^+}{\partial y}\frac{\partial \bar{v}_x}{\partial v_x^+} = \frac{v_\tau^2}{\nu} = \frac{\tau_w}{\rho\nu}$$

$$\frac{1}{\rho}\tau_w = (\nu + \nu_t)\frac{\partial v_x^+}{\partial y^+}\frac{\tau_w}{\rho\nu}$$

$$\boxed{\frac{\nu_T}{\nu} = \left(\frac{\partial v^+}{\partial y^+}\right)^{-1} - 1}$$

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See T&K §10.2.1

The turbulent viscosity

$$\frac{\nu_T}{\nu} = \left(\frac{\partial v^+}{\partial y^+}\right)^{-1} - 1$$

Given a wall velocity profile, we may obtain ν_{turb}! Then we can get D_{turb} from $\mathrm{Sc}_{\text{turb}}$.

von Karman velocity profile (model - one of many)

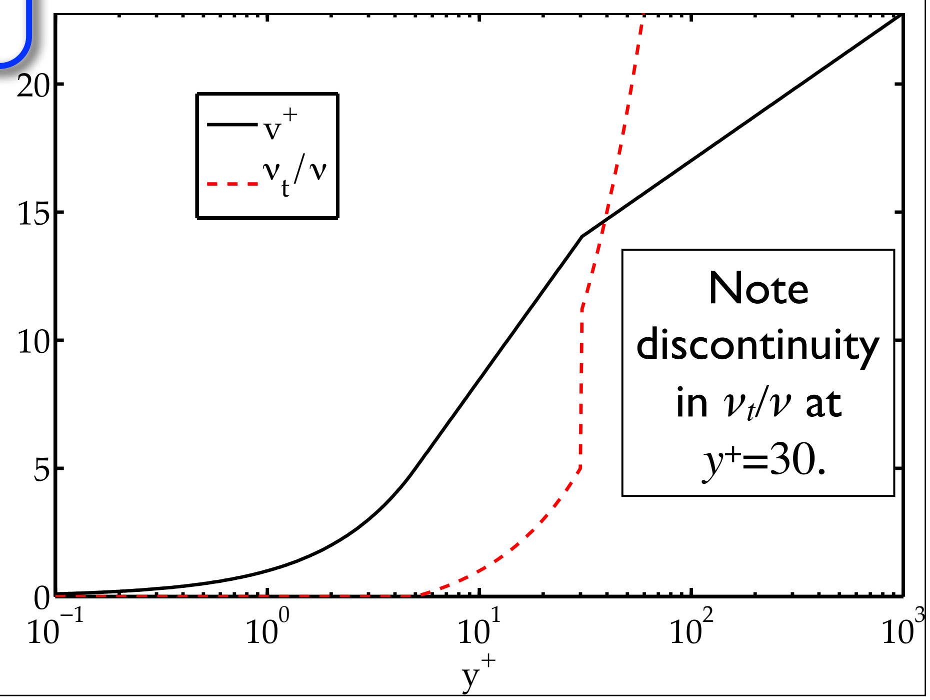
Visous sublayer: $v^+ = y^+ \qquad 0 \le y^+ < 5$

Buffer zone: $v^+ = 5 \ln y^+ - 3.05 \qquad 5 \le y^+ < 30$

Turbulent core: $v^+ = 2.5 \ln y^+ + 5.5 \qquad 30 \le y^+$

$$\begin{aligned} \nu_{\text{turb}}/\nu &= 0 & 0 \le y^+ < 5 \\ \nu_{\text{turb}}/\nu &= y^+/5 - 1 & 5 \le y^+ < 30 \\ \nu_{\text{turb}}/\nu &= y^+/2.5 - 1 & 30 \le y^+ \end{aligned}$$

$y^+ > 30$ is typically just called "fully turbulent" where ν is ignored.



See T&K §10.2.1

Another way to get ν_{turb}.

Prandtl's mixing length hypothesis:

$$\nu_{\text{turb}} = \ell_m^2 \left| \frac{\partial \bar{v}_x}{\partial y} \right|$$

$$\ell_m^+ \equiv \ell_m v_\tau = \lambda_p y^+$$

See §10.2.1 in T&K or §7.3.3 in S.B. Pope's book for derivation...

$$\frac{\nu_T}{\nu} = \left(\frac{\partial v^+}{\partial y^+} \right)^{-1} - 1 \qquad \frac{\partial v^+}{\partial y^+} = \frac{-1 + \sqrt{1 + 4(\ell_m^+)^2}}{2(\ell_m^+)^2}$$

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Recap

- Model entire turbulence process
  - No direct resolution of the flow field (*e.g.* no DNS, LES, RANS)
  - Turbulent mixing → increased gradients → enhanced diffusion at small scales
  - Turbulent mixing affects all species equally
    - Multicomponent effects only present at smallest scales
    - If $L \gg \eta$, then molecular mixing is "unimportant" relative to turbulent mixing.
      - Multicomponent effects still exist, but are mixed out rapidly and don't affect the "large" scales.
      - At walls, multicomponent effects become important since $L/\eta \to 1$.

Recall: $\mathrm{Sc}_{\mathrm{turb}} = \dfrac{\nu_{\mathrm{turb}}}{D_{\mathrm{turb}}}$ $\mathrm{Pr}_{\mathrm{turb}} = \dfrac{c_p \mu_{\mathrm{turb}}}{\lambda_{\mathrm{turb}}}$

Therefore, if we prescribe $\mathrm{Sc}_{\mathrm{turb}}$, $\mathrm{Pr}_{\mathrm{turb}}$, then the problem becomes determining ν_{turb}, (or μ_{turb}).

No multicomponent effects for turbulent diffusion terms!

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