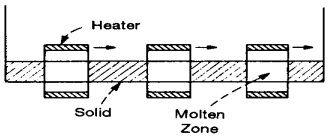
Zone Refining

$D_L \approx 8 \cdot 10^{-4} \cdot \frac{cm^2}{s}$

$U \approx 0.01 \cdot \frac{cm}{s}$

$C_{oO} \approx 0.001$

$C_{oFe} \approx 0.0001$



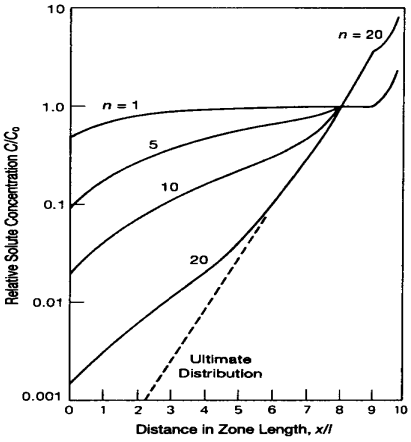
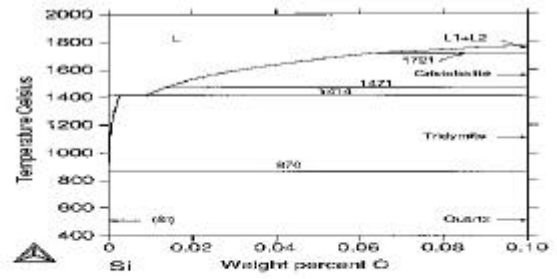


Figure 7. Relative solute concentration in a germanium ingot that has been zone refined with n *passes.*

$$\frac{C_S}{C_o} = \frac{1}{2}\left\{1 + \operatorname{erf}\sqrt{\frac{Ux}{4D_L}} + (2k-1)\exp\left[-k(1-k)\frac{Ux}{D}\right]\operatorname{erfc}\left[(2k-1)\sqrt{\frac{Ux}{4D_L}}\right]\right\} \quad (18)$$

$C_s(x,k,C_o) \approx \frac{C_o}{2}\left(1 - \operatorname{erf}\left(\sqrt{\frac{U\cdot x}{4\cdot D_L}}\right) - (2\cdot k - 1)\cdot\exp\left(k\cdot(1 - k)\cdot\frac{U\cdot x}{D_L}\right)\cdot\operatorname{erfc}\left((2\cdot k - 1)\cdot\sqrt{\frac{U\cdot x}{4\cdot D_L}}\right)\right)$

Phase Diagrams



Si-O Crystal Structure Data

Phase	Pearson Symbol	Structur Prototype Bericht	Mineral
(Si)	cF8	A4	Diamond
Cristobalite	tP12		
Tridymite	hP12		
Quartz	hp9		

$k_O \approx \frac{0.002}{0.01}$

$C_{oO} := 1 \cdot 10^{-3}$

$C_s(0\,cm)\,k_O\,C_{oO} := 2 \cdot 10^{-4}$ First Pass

$C_s(0\,cm)\,k_O\,C_s(0\,cm)\,k_O\,C_{oO} := 4 \cdot 10^{-5}$ Second Pass

C_o=solute concentration in melt or of solid on first pass

$$C_o = \int_0^{x+L} C_s(x)\,dx - \int_o^{x-L} kC_L(x)\,dx$$

$$C_L(x)\,k\,C_o := C_o\left(1 - \frac{1-k}{k}\exp\left(-\frac{U\,x}{D_L}\right)\right)$$

$L := 0.1\,cm$ Length of Heater

$$C_{O'}(x) := \int_{0\,cm}^{x-L} C_s(s)\,k_O\,C_{oO}\,ds + \int_{0\,cm}^{x-L} k_O\,C_L(s)\,k_O\,C_{oO}\,ds$$

Concentration Profile, Pass 1 and 2