CHAPTER 10

Gas Dispersion and Gas Interchange in Bubbling Beds

As the result of the movement of solids and the bubbling action, the fluidizing gas passes in a complex manner through the bed. Prediction of bed behavior for various operations, particularly catalytic reactions, requires knowing how the gas passes through the bed, its dispersion, and its interchange between bubble and emulsion phases. This chapter deals with these matters. We deal first with the dispersion phenomena, both vertical and horizontal, and conclude with measures of gas interchange between regions.

Dispersion of Gas in Beds

Vertical and horizontal dispersion of gas in bubbling beds has been examined using a variety of steady and unsteady state tracer techniques, and the results of these studies have been interpreted in several ways:

- By running steady state gas tracer experiments and adopting a diffusion-type model with vertical and horizontal dispersion coefficients \( D_{v} \) and \( D_{h} \) to represent the deviation of flow from the ideal of plug flow in the bed.
- By running stimulus-response experiments and fitting a diffusion-type model with dispersion coefficient \( D_{v} \) to the response curve.
- By injecting tracer bubbles into incipiently fluidized beds, following the loss of tracer from these bubbles and thereby finding the interchange coefficient between phases.
- By running stimulus-response experiments and fitting these results with a two-region model that includes gas interchange between regions.

Steady State Tracer Studies

The steady state tracer experiment, first used by Gilliland and Mason [1] and sketched in Fig. 1, introduces a steady flow of tracer gas at a horizontal plane in a tall, narrow fluidized bed and measures the upstream diffusion of the tracer.
CHAPTER 10 — Gas Dispersion and Gas Interchange in Bubbling Beds

Dispersal of Gas in Beds

FIGURE 1
Steady state experiment for finding the vertical dispersion coefficient of gas, $D_{GV}$, in a fluidized bed.

Figure 2 shows typical experimental results on the backmixing of gas. Here $C_A$ is the mean concentration of tracer at level $z$ in the bed (upward is +) and $C_{AI}$ is the concentration at the injection plane $z_I$. Note the greatly increased back mixing of gases that are adsorbed by the porous bed solids and are thereby carried down the bed by these solids.

When represented by the dispersion model, the differential equation for this vertical dispersion process is

$$D_{GV} \frac{d^2 C_A}{dz^2} - u_0 \frac{d C_A}{dz} = 0$$

(1)

and with reasonable boundary conditions its solution is

$$C_A = \exp \left[ - \frac{u_0 (z - z_I)}{D_{GV}} \right]$$

(2)

Values of $D_{GV}$ thus determined by Yoshida et al. [4] are shown in Fig. 3(a). They indicate that $D_{GV}$ is approximately proportional to $u_0$. Miyachi et al. [7] summarize the reported data on the effect of bed size on $D_{GV}$ in Fig. 4. Figures 3(b) and 4 also present $D_{GV}$ values determined by the stimulation-response method, considered in the next section.

A second steady state technique introduces a continuous stream of tracer at a point in the bed, usually at the axis, while measuring this tracer at various neighboring positions in the bed. Solving the diffusion equation for cylindrical coordinates with the appropriate boundary conditions gives the horizontal dispersion coefficient for gas, $D_{GH}$. Table 1 and Fig. 5(a) give values of $D_{GH}$ found this way for fine particle systems. Note that as gas flows near to and below $u_{inf}$, the measured $D_{GH}$ in these fine particle beds is close to the molecular diffusivity of the tracer gas.

FIGURE 2
Vertical back-mixing profiles of tracer gas introduced as in Fig. 1. Note the difference between the profiles of adsorbed and nonadsorbed gases: (a) $d_I = 0.3 \text{ m}$, $d_A = 145 \mu m$, from Nguyen and Potter [2]; (b) $d_I = 0.1 \text{ m}$, $d_A = 70 \mu m$, from Bohle and van Swaaij [3].

FIGURE 3
Vertical dispersion of gas in fluidized beds: (a) steady backmixing experiments, circular points for microspherical catalyst $d_A = 150 \mu m$; triangular points for FCC catalyst 60 \mu m; adapted from Yoshida et al. [4]; (b) stimulation-response experiments by Schäfer [5], adapted from Zuijderweg [6].
Jovanović et al. [17] studied the horizontal spread of gas in large particle Geldart BD and D beds by introducing tracer gas continuously at one location on the distributor plate of a two-dimensional bed and noting where it appeared at the bed surface. The time-averaged tracer concentration curve was found to be bell-shaped with its maximum located directly above the tracer feed. This suggests a diffusional spread of tracer, from which $D_{gh}$ can be found. Experiments, summarized in Fig. 5(b), show that horizontal tube arrays enhance dispersion at low gas velocity. However, at higher velocity $D_{gh}$ levels off for beds with tube bundles but increases steadily in beds without internals.

Examining these results more carefully, they noted instantaneous tracer readings as shown in Fig. 6(a), in which the tracer missed the probe completely much of the time. This behavior cannot be explained by simple diffusion theory, so they attributed this phenomenon to the meandering of a plume of tracer, somewhat as a flickering candle flame. This is sketched in Fig. 6(b). Developing a model of this sort, they found the following expression for the horizontal dispersion coefficients:

$$D_{gh} = D_{gm} + D_{gt}$$

where $D_{gm}$ represents the meandering of the plume and $D_{gt}$ represents the turbulent or actual intermixing of gases about the axis of the plume. By measuring the time-average spread of tracer and the spread of the root mean square concentration of tracer, Jovanović et al. showed how to evaluate the individual dispersion coefficients. For 0.46-m high beds of 4-mm solids at gas velocities of 0.5–2 m/s, they found the following values:

$$\begin{array}{c|cc}
\text{Without tubes} & D_{gh} (m^2/s) & D_{gt} (m^2/s) \\
\hline
0.008–0.030 & 0.004 \\
\end{array}$$

With tubes

$$\begin{array}{c|cc}
\text{With tubes} & D_{gh} (m^2/s) & D_{gt} (m^2/s) \\
\hline
0.020–0.035 & 0.015–0.018 \\
\end{array}$$

**Figure 4**
Effect of bed diameter on the vertical dispersion of gas in beds of Geldart A solids (FGC catalyst); data taken mainly from Miyachi et al. [7].

**Table 1** Horizontal Dispersion Coefficient of Gas $D_{gh} (m^2/s)$ in Fluidized Beds

<table>
<thead>
<tr>
<th>Observer</th>
<th>$d_b$ (m)</th>
<th>Particles</th>
<th>$d_p$ (μm)</th>
<th>Gas velocity (m/s)</th>
<th>Tracer</th>
<th>$D_{gh}$ (m^2/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jovanović et al. [17] (1986)</td>
<td>0.075</td>
<td>Sand</td>
<td>75</td>
<td>$u_g/\sqrt{f} = 0.1–0.3$</td>
<td>H$_2$</td>
<td>$0.4–2.2 \times 10^{-2}$ (75 μm)</td>
</tr>
<tr>
<td>Hiraki et al. [16] (1968/69)</td>
<td>0.20</td>
<td>Cat.</td>
<td>175</td>
<td>$u_g = 0.1–0.3$</td>
<td>H$_2$</td>
<td>$1–5 \times 10^{-3}$ (175 μm)</td>
</tr>
</tbody>
</table>

**Figure 5**
Horizontal dispersion of gas in beds: (a) near $u_mf$ in beds of fine solids; $d_b = 0.2$ m, microospherical catalyst, $d_p = 150$ μm; from Hiraki et al. [16]; (b) effect of tube array in beds of coarse particles; $0.46 \times 0.13$ m, $u_mf = 0.73–1.83$ m/s; from Jovanović et al. [17].

**Figure 6**
Horizontal spread of tracer in large particle beds. (a) Fluctuating tracer concentration at a point on the bed surface. This suggests a meandering plume of tracer. (b) Main aspects of the meandering plume model.
These results indicate that in the absence of bed internals the meandering plume is the main mechanism for horizontal spread of gas, whereas in beds with tube bundles actual turbulent mixing dominates.

Note that the turbulent dispersion coefficient $D_L$ is the pertinent measure of contacting or mixing, and the meandering coefficient $D_m$ does not contribute to the actual mixing of gas from the standpoint of chemical reaction. Thus, for coarse particles, tube-filled beds have a greater "useful" horizontal dispersion.

**Stimulus-Response Studies**

The stimulus-response technique has been used extensively to explore the flow behavior of gas in fluidized beds. A brief outline of the method can be found in Chap. 6 of [18], with a more detailed explanation in [19].

Figure 7 shows typical output curves obtained from pulse and step inputs. Thus, Figs. 7(a) and (b) show the response for the ideal of plug flow and for the ideal of mixed flow (or backmix flow). Here the mean residence time of fluid in the bed is

$$ t = \frac{\text{volume of void space in the bed}}{\text{volumetric feed rate of gas}} = \frac{\varepsilon V}{u_0} $$  \hspace{1cm} (4)

and the concentration measure is normalized such that the area under the pulse-response curve is unity. These normalized curves are called the $E(t)$ curve for the pulse response, and the $F(t)$ curve for the step response. In addition, the $E(t)$ and $F(t)$ curves are related. Thus, at any time $t$ after introduction of the tracer,

$$ \frac{dF(t)}{dt} = E(t) \quad \text{or} \quad F(t) = \int_0^t E(t) \, dt $$  \hspace{1cm} (5)

Figures 7(c) and (d) compare the response curves of ordinary (nonadsorbed) tracer with tracer that is adsorbed on the bed solids. Note that the mean of the ordinary tracer curve is at $\bar{t}$. However, the absorbing tracer is held back by the solids and leaves the bed later than expected.

If the extent of adsorption is represented by an equilibrium constant defined by

$$ m = \frac{\text{concentration of tracer in the solid}}{\text{concentration of tracer in the gas}} $$  \hspace{1cm} (6)

then for a bed of adsorbing solids

$$ t < \bar{t}_{\text{measured}} < (1 + m)\bar{t} $$  \hspace{1cm} (7)

and if equilibrium is rapidly established, then

$$ \bar{t}_{\text{measured}} \rightarrow (1 + m)\bar{t} $$  \hspace{1cm} (8)

Figure 8 shows that the measured response curves for nonadsorbed tracer gases.

**Figure 7**

(a) and (b) Pulse and step response for plug flow and mixed flow of gas. (c) and (d) Comparison of response curves for adsorbed and nonadsorbed tracer gases.

**Figure 8**

Residence time distribution curves for nonadsorbed tracer gas: (a) $E(t)$ and $F(t)$ curves for the regenerator of a commercial FCC unit; from Daukswerts et al. [20]; (b) points A: $F(t)$ curve, $d_l = 0.076$ m, $L = 0.114$ m, $u_0 = 0.134 - 0.305$ m/s, from Gilliland and Mason [1]; solid curve: $F(t)$ curve, $d_l = 1.53$ m, $u_0 = 0.244$ m/s, from May [12].
gas in both small and large fluidized beds lies somewhere between plug flow and mixed flow. Figure 9 shows that adsorption of a component of the gas stream can result in serious holdback of that material in the bed. Reviewing the reported data such as shown in Figs. 8 and 9 for a variety of fluidizing conditions, we find that gas is closer to plug flow in larger particle beds and in larger-diameter beds.

The one-dimensional diffusion-type model often reasonably represents flows that do not deviate much from plug flow, and its differential equation relating the response curve with the dispersion coefficient is

$$\frac{dC_2}{dt} = D_{CV} \cdot \frac{1}{u_0} \cdot \frac{\partial^2 C_2}{\partial z^2} - \frac{C_2}{u_0}$$

Solving for a pulse input gives a simple expression for $D_{CV}$ in terms of $\sigma^2$, the variance of the E(t) curve, as follows:

$$\sigma^2 = 2\pi \left( \frac{D_{CV}}{u_0} \right)$$

More generally, for any “one-shot” upstream tracer input into the bed, the increase in variance of the response curves between outputs 1 and 2, $\sigma_2^2$ and $\sigma_2^2$, respectively, is related to $D_{CV}$ by

$$\sigma_2^2 - \sigma_1^2 = 2(u_3 - i_1) \left( \frac{D_{CV}}{u_0} \right)$$

where $i_1$ and $i_2$ are the means of these output curves, measured from any starting time.

The above expressions and similar expressions can be used to obtain $D_{CV}$ from experiment. Examples of such data reported by Zuidervogel [6], using Schügerl’s experimental results [5], are shown in Fig. 3(b). Miyoshi et al. [7] compared values of $D_{CV}/u_0$ obtained by several workers, and these are shown as the solid data points in Fig. 4. The open points in this figure are from steady state experimentation mentioned earlier.

For gaseous components that can be adsorbed by the bed solid, the vertical dispersion of tracer gas should also include the material carried about the bed by these solids. Miyoshi and Kaji [11] assumed that equilibrium existed between gas and solid at all points in the bed and came up with the following expression to account for adsorbed gases:

$$D_{CV} = \frac{D_{CV_{nond}}}{m} + \frac{mD_{CV}}{\text{m}}$$

Thus, in addition to the diffusive flux of gas, they included the term $m$ to account for the diffusive flux of the gaseous component while it is adsorbed by the solid. Table 2 shows the values of $m$ found.

### Definitions of Gas Interchange

First we introduce the various measures of gas interchange.

**Interchange Coefficients $K_{A_{12}}$, $K_{A_{13}}$, and $K_{A_{14}}$.** Consider the removal of material A from a bubble of volume $V_b$. Based on unit volume of bubble, the interchange coefficient between bubble and cloud ($K_{A_{12}}$), between cloud and emulsion ($K_{A_{13}}$), and the overall coefficient between bubble and emulsion ($K_{A_{14}}$) can be defined by the rate equations

$$- \frac{1}{V_b} \frac{dN_{A_{12}}}{dt} = -u_b \frac{dC_{A_{12}}}{dz} = K_{A_{12}}(C_{A_{12}} - C_{A_{13}})$$

$$= K_{A_{13}}(C_{A_{13}} - C_{A_{14}})$$

where the $K$ values have dimensions of $s^{-1}$, and $C_{A_{12}}$, $C_{A_{13}}$, and $C_{A_{14}}$ are the mean concentrations of gaseous component A in the bubble, in the gas cloud and emulsion, respectively. The relationship between

<table>
<thead>
<tr>
<th>Tracer</th>
<th>$D_{CV}(m^2/s)$</th>
<th>$u_0 (m/s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>0.6</td>
<td>0.03</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>4.5</td>
<td>0.10</td>
</tr>
<tr>
<td>CF$_3$</td>
<td>10</td>
<td>0.15</td>
</tr>
</tbody>
</table>

$D_{CV}$ (m$^2$/s), taken from Miyoshi and Kaji [11].

$k_0$ = 0.079 $m$; $d_p$ = 38 $\mu$ m.
interchange coefficients is then
\[
\frac{1}{K_{be}} = \frac{1}{K_{bc}} + \frac{1}{K_{ce}}
\]  
(14)

From a physical standpoint, the interchange coefficient \(K_{ce}\) can be looked upon as a flow of gas from bubble to emulsion with an equal flow in the opposite direction:
\[
K_{ce} = \left[ \frac{\text{volume of gas going from bubbles}}{\text{time}} \right] \left[ \frac{\text{to emulsion or from emulsion to bubbles}}{\text{volume of bubbles in the bed}} \right], \quad [\text{s}^{-1}]
\]  
(15)

The two other interchange coefficients, \(K_{bc}\) and \(K_{ce}\), have similar meanings. These coefficients are sometimes called the crossflow ratios.

**Crossflow Ratio \(X_b\).** The gas interchange between bubble and the rest of the bed may also be expressed as a dimensionless crossflow ratio, defined as
\[
X_b = \frac{\text{number of times the bubble gas is replaced}}{\text{as the bubble passes through the bed}}.
\]  
[−]

(16)

Note that for uniform bed conditions \(K\) is independent of bed height and \(X_b\) varies linearly with bed height.

**Mass Transfer Coefficient from Bubble to the Dense Region, \(k_{be}\).** The net flux of tracer \(A\), \(N_{be}(m/s)\), from a bubble of volume \(V_b\) and surface \(S_{be}\) is given by
\[
-\frac{dN_{be}}{dt} = -n_b V_b \frac{dC_{A_b}}{dx} = S_{be} k_{be}(C_{A_b} - C_{A_e})
\]  
(17)

If \(n_b\) is the bubble-emulsion interfacial area per unit volume of bed, then the volumetric mass transfer coefficient is
\[
k_{be} \delta_b = k_{be} \frac{\delta}{\delta_b}, \quad [\text{s}^{-1}]
\]  
(18)

**Interrelationship between Transfer Coefficients.** By comparing the above equations, we find
\[
K_{be} = \frac{S_{be}}{V_b} = \frac{6}{d_b} k_{be} = \frac{k_{be} n_b}{\delta}, \quad [\text{s}^{-1}]
\]  
(19)

For vigorously bubbling beds where \(\delta = u_d/u_b\) (see Eq. (6.29)), we also have
\[
K_{bc} \approx k_{be} \frac{u_b}{u_d}
\]  
(20)

**Experimental Methods**

There are two experimental approaches: first, to analyze the loss of tracer gas from single bubbles injected into a fluidized bed otherwise at minimum fluidizing conditions and, second, to analyze tracer concentrations in ordinary bubbling beds. Also, this whole section applies to fine particle beds containing bubbles with their thin clouds of recirculating gas, because it is here where bubble-emulsion transfer is slow enough to cause difficulties.

**Single-Bubble Method.** Consider a single clouded bubble containing tracer \(A\) at concentration \(C_{A_b}\) injected at level \(z_i\) into a fluidized bed that contains \(A\) at \(C_{A_e}\). Normally \(C_{A_e} = 0\). With the following boundary condition for the bubble gas,
\[
C_{A_b} = C_{A_i} \quad \text{at} \quad z = z_i
\]

Equation (13) integrates simply to give
\[
C_{A_b} - C_{A_e} = \exp \left[ - \frac{K_{be}(z - z_i)}{u_b} \right]
\]  
(21)

\(K_{be}\) is then found by measuring the changing concentration of \(A\) in the rising bubble.

**Bubbling Bed Method.** Pulse- or step-response tracer measurements put in the framework of the two-region model of Chap. 6 can be used to yield values for the interchange coefficient. A simple version of this model for fast clouded bubble beds with either upflow or downflow of emulsion gas is shown in Fig. 10. The differential equations representing the movement of tracer introduced

![FIGURE 10](image)

Two-region model to obtain the gas interchange coefficient between bubble and the rest of the bed: (a) upflow of emulsion; (b) downflow of emulsion.
uniformly across the bottom of the bed are

$$\frac{dC_{Ah}}{dt} + u_b \frac{dC_{Ah}}{dz} = K_{be}(C_{Ah} - C_{Ae})$$

$$\frac{dC_{As}}{dt} + \frac{u_s}{e_s} \frac{dC_{As}}{dz} = \frac{\delta}{1 - \delta} K_{be}(C_{Ah} - C_{Ae})$$

(22)

where

$$u_e = \delta u_b + (1 - \delta) u_s$$

(23)

Knowing $u_b$, $u_s/e_s$, and $\delta$ and matching the measured tracer response to the curves derived from this model will then yield $K_{be}$.

Dayan and Levenspiel [22] evaluated the $E(t)$ curves for both up and down emulsion flow by various methods—characteristics, Riemann function, Monte Carlo and Markov chain—and Fig. 11 displays some of their results. Compare these with the $E(t)$ curve of Fig. 8(a) for a very large commercial FCC regenerator.

De Groot [32] and van Deemter [34] also developed this type of model. In addition, van Deemter made a parametric comparison between the above model and a more complicated version that included a diffusion term. He concluded that the simpler model was more realistic for beds where the scale of the circulation currents was comparable to the dimensions of the bed.

Tamazaki and Miyachi [25] and Morooka et al. [36] extended this method to account for tracer gases that are adsorbed on the bed solids.

**Experimental Findings on Interchange Coefficients**

Figure 12 shows values of $K_{be}$ obtained from the single injected bubble technique. These values are only for nonadsorbed or negligibly adsorbed gases.

For tracer gas adsorbed on the bed solids, the measured $K_{be}$ values are much higher than the values given in Fig. 12. For example, for the adsorption by microspherical catalyst particles of water vapor from bubbles of humid air, Wakabayashi and Kunii [31] found that

$$K_{be} = 15-10 \text{ s}^{-1} \quad \text{for } d_b = 0.04-0.11 \text{ m}$$

These values are one order of magnitude higher than for nonadsorbed tracer gas, as may be seen from Fig. 12. Similar enhancement was found by Toci et al. [32] with Freon-12 on activated alumina and by Rietema et al. [33,34] with hydrocarbon tracer on spent cracking catalyst.

Experimental gas interchange findings obtained by the bubbling bed method are presented in Figs. 13 and 14. Figure 13 shows that the volumetric flux increases with gas flow rate and with increased adsorption characteristics of the system, and Fig. 14 shows a gradual decrease of flux in larger beds. This is probably a consequence of having faster and larger bubbles in these beds. As can be seen in these two figures, the volumetric mass transfer coefficient $k_{be}d_b$ was the measure used by experimenters to evaluate the interchange rate, not $K_{be}$. However, Eqs. (19) and (50) relate these quantities. (Note that the lines in Fig. 13(b) come from an analysis developed in the next chapter (see Example 11.2), so ignore them for now.)
Estimation of Gas Interchange Coefficients

From the detailed behavior of gas about bubbles we can estimate these interchange coefficients. First consider the interchange between bubble and cloud for fast clouded bubbles, \( u_{sw} > 5 u_{mf}/e_{mf} \). This involves both bulk flow and diffusion across the boundary. So, referring to Fig. 15, we have for the removal of tracer A in a single rising bubble

\[
- \frac{dN_{Ab}}{dt} = (q + k_{bc} S_{bc})(C_{Ab} - C_{Ac})
\]  

(24)

where \( q \) is the volumetric gas flow into or out of a single bubble and \( k_{bc} \) is the mass transfer coefficient between bubble and cloud (see Eq. (17)). From the

\[
q = \frac{3\pi}{4} u_{mf} d_{B}^{2}, \quad \text{[m}^{3}/\text{s}]\]

(35)

Assuming a spherical cap bubble of \( \theta = 100^\circ \) and the Higbie penetration model with diffusion limited to a thin layer at the interface, Davidson and Harrison [37] derived the following expression for the mass transfer coefficient between bubble and cloud:

\[
k_{bc} = 0.975 \theta^{1/3} \left( \frac{g d_{B}}{d_{B}} \right)^{1/4}, \quad \text{[m/s]}\]

(36)

Substituting these two expressions in Eq. (24) and matching with Eq. (13) gives the interchange coefficient between bubble and cloud:

\[
K_{bc} = 4.5 \left( \frac{u_{mf}}{d_{B}} \right)^{1/3} + 5.68 \left( \frac{d_{B}^{1/3} \theta^{1/4}}{d_{B}^{1/4}} \right), \quad \text{[m/s]}
\]

(27)

Next, estimate the coefficient between cloud and emulsion. Because there is no flow of gas between these regions, diffusion will be the only acting mechanism, so we have

\[
- \frac{dN_{Ac}}{dt} = S_{ac} k_{cc}(C_{Ac} - C_{Aa})
\]

(35)
where $k_{oe}$ is the mass transfer coefficient between cloud and emulsion and $S_{oe}$ is the cloud-emulsion interfacial area of a bubble. Since the exposure time is the same for all elements of interface moving from the top to the bottom of the bubble, this process is best represented by the Higbie penetration model. Analogous to the contacting of a bubble by liquid, the characteristics of this system are equivalent to the contacting of a vertical cylinder with the same diameter and height as the spherical cloud (see Higbie [38] for details). Thus

$$k_{oe} = \left( \frac{4 \sigma_{oe} e_{mf}}{\pi} \right)^{1/2}, \quad [\text{m/s}]$$

(29)

For these bubbles with thin clouds (see Fig. 5.3) we can take

$$d_e \approx d_b \quad \text{and} \quad \frac{S_b}{V_b} \approx \frac{6}{d_b}$$

(30)

Thus the exposure time of an element of bubble surface with the emulsion is

$$t = \frac{d_e}{u_{br}} \approx \frac{d_b}{u_{br}}$$

(31)

Inserting Eqs. (30) and (31) into Eq. (29) and matching with Eq. (19) gives

$$k_{oe} = \left( \frac{4 \sigma_{oe} e_{mf} u_{br}}{\pi d_b} \right)^{1/2} \frac{S_b}{V_b} \approx 6.77 \left( \frac{\sigma_{oe} e_{mf} u_{br}}{d_b^2} \right)^{1/2}$$

(32)

Utilizing the stream function for gas around a spherical cap bubble, Chiba and Kobayashi [29] solved the fundamental equation governing diffusion through the cloud-emulsion interface. For the special case of a spherical bubble, without assuming that $d_e \approx d_b$, but requiring that $\Theta_e = \Theta_{mf}$ they their analytical result reduces precisely to Eq. (32).

Results of experiments in fine particle systems, such as shown in Fig. 5(a), suggest that the effective diffusion coefficient of gas in the emulsion phase of a fluidized bed is better approximated by

$$\Theta_e \approx \Theta$$

(33)

where $\Theta$ is the diffusion coefficient of the gas alone. With this approximation and with $u_{br} = 0.711(gd_b)^{1/2}$. Eq. (32) becomes

$$K_{oe} = 6.77 \left( \frac{\sigma_{mf}(0.711)(g d_b)^{1/2}}{d_b^2} \right)^{1/2} \approx 6.77 \left( \frac{\sigma_{mf} u_{br}}{d_b^2} \right)^{1/2}, \quad [\text{s}^{-1}]$$

(34)

Comments about the Measured $K_{be}$ by the Bubble Injection Technique. Consider the period just after injection of a tracer-laden bubble into a tracer-free bed. The cloud has little or no tracer; hence, measurements of $K_{be}$ under these unsteady state conditions really reflect the bubble-cloud inter-

change. As a result, close to the time of bubble injection

$$K_{be, \text{measured}} \approx K_{bc}$$

Thus, although Eq. (14) properly gives

$$\frac{1}{K_{bc}} = \frac{1}{K_{be}} + \frac{1}{K_{oo}}$$

the short-time measured $K_{be}$ will lie between the true $K_{be}$ and the true $K_{bc}$, the closeness to one or other of these values depending on the experimental conditions.

Figure 12 shows that the measured $K_{bc}$ values do lie between the predicted $K_{bc}$ from Eq. (27) and $K_{be}$ obtained by combining Eqs. (27) and (34) according to Eq. (36).

We emphasize that these interface expressions only properly represent nonabsorbed gases. For kinetic processes, such as sublimation or solid-catalyzed gas-phase reactions in beds of porous particles, Eq. (14) or (36) should not be used directly. The next chapter shows how these interphase coefficients for nonabsorbed gases can be incorporated into a "bubbling bed" model that accounts for the kinetic processes involving gaseous species that are adsorbed or somehow processed by the bed solids.

Example 1

Estimate $K_{bc}$ and $K_{oo}$ for the operating conditions of Fig. 12.

Data

<table>
<thead>
<tr>
<th>Fine particles</th>
<th>$u_{mf} = 0.01 \text{ m/s}, \sigma_{mf} = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracer gas, ozone</td>
<td>$\Theta = 2 \times 10^{-5} \text{ m}^2/\text{s}$</td>
</tr>
<tr>
<td>Tracer gas, helium</td>
<td>$\Theta = 7 \times 10^{-5} \text{ m}^2/\text{s}$</td>
</tr>
<tr>
<td>Coarser particles</td>
<td>$u_{mf} = 0.045 \text{ m/s}, \sigma_{mf} = 0.5$</td>
</tr>
<tr>
<td>Tracer gas, ozone</td>
<td>------------------------------------------</td>
</tr>
</tbody>
</table>

Solution

Calculate $K_{bc}$. For the fine particles with ozone tracer, Eq. (27) gives

$$K_{bc} = \frac{4.5 \times 0.01}{4.5} + 5.95 \left( \frac{2 \times 10^{-5}}{\Theta} \right)^{1/2} \left( g d_b \right)^{1/4}$$

(35)

$$\frac{1}{K_{bc}} = \frac{1}{K_{be}} + \frac{1}{K_{oo}}$$

(36)

This equation is shown as line 1 in Fig. 12. Similarly,

$K_{bc}$ for fine particles and helium: line 2 in Fig. 12

$K_{bc}$ for coarser particles and ozone: line 3 in Fig. 12

Calculate $K_{oo}$. Equation (14) gives $K_{oo}$ as

$$\frac{1}{K_{oo}} = \frac{1}{K_{bc}} + \frac{1}{K_{bc}}$$

(37)

$K_{bc}$ has been evaluated above. To evaluate $K_{oo}$, apply Eq. (34), which gives

$$K_{oo} = 6.77 \left( \frac{2 \times 10^{-5}}{\Theta} \right)^{1/2} \left( 0.045 \right)^{1/2} \left( g d_b \right)^{1/2}$$

(38)
CHAPTER 10.—Gas Dispersion and Gas Interchange in Bubbling Beds

Replacing Eqs. (i) and (ii) in (i) gives \( K_{\text{be}} \). Thus, for long residence time we obtain

\[
K_{\text{be}} \quad \text{for fine particles and ozone:} \quad \text{line 4 in Fig. 12} \\
K_{\text{be}} \quad \text{for fine particles and helium:} \quad \text{line 5 in Fig. 12}
\]

EXAMPLE 2

In fluidized bed processes where the rate-controlling step is the transfer of gaseous component from bubble gas to the bed solids, some workers propose models in which the bubble-cloud interchange is rate controlling (see Fig. 16(a)), others take the cloud-emulsion interchange to be rate controlling (see Fig. 16(b)), and still others say that both should be considered (see Fig. 16(c)).

From the values of \( K_{\text{be}} \) and \( K_{\text{ce}} \) for the ammonia-air system (\( \beta = 0.69 \) cm²/s) in beds of nonabsorptive particles (\( \mu_{\text{rel}} = 1 \) cm/s, \( \epsilon_{\text{rel}} = 0.5 \)), determine the relative importance of these two transfer resistances and determine what error is introduced when the minor resistance is ignored.

(a) Consider beds with 5-cm bubbles.
(b) Consider high-velocity operations where beds have 15-cm bubbles.

SOLUTION

From Eq. (27) we have

\[
K_{\text{be}} = 4.5 \left( \frac{a_b}{d_b} \right) \left[ \frac{0.69}{d_b^2} \right]^{1/2} \left[ \frac{980}{d_b^2} \right]^{1/4} = 4.5 \frac{a_b}{d_b^2} \left[ \frac{980}{d_b^2} \right]^{1/4}
\]

(i)

From Eq. (34)

\[
K_{\text{ce}} = 6.77 \left[ \frac{0.69}{d_b^2} \left( 0.5 \frac{980}{d_b^2} \right)^{1/2} \right]^{1/2} = 18.76 \frac{a_b}{d_b^2}
\]

(ii)

Comparing these coefficients for 5-cm and 15-cm bubbles, we obtain the following information:

<table>
<thead>
<tr>
<th>( d_b ) (cm)</th>
<th>( K_{\text{be}} )</th>
<th>( K_{\text{ce}} )</th>
<th>( K_{\text{be}} ) from Eq. (14)</th>
<th>Error in ( K_{\text{be}} ) when minor resistance is ignored</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.54</td>
<td>2.51</td>
<td>1.62</td>
<td>55% high</td>
</tr>
<tr>
<td>15</td>
<td>1.22</td>
<td>0.84</td>
<td>0.452</td>
<td>51% high</td>
</tr>
</tbody>
</table>

Problem

A comparison of Eqs. (i) and (ii), as well as the results of the above table, shows that the cloud-emulsion interchange provides the major resistance. However, both resistances are of the same order of magnitude. Therefore, in modeling these transfer processes, one should consider both resistances if this will not unduly complicate matters.

EXAMPLE 3

Drinkenburg and Rietema [33] reported finding \( K_{\text{be}} = 0.028 - 0.05 \) m/s for adsorbed hydrocarbon tracer in a 0.9 m ID fluidized bed of cracking catalyst. Compare this with the interchange rate for nonadsorbed tracer gas.

Data

\[
\mu_{\text{be}} = 0.30 \text{ m/s}, \quad \mu_{\text{g}} = 0.13 \text{ m/s}, \quad m = 7, \quad \epsilon_{\text{rel}} = 0.5
\]

\[
\mu_{\text{rel}} = 0.0018 \text{ m/s}, \quad \beta = 9 - 22 \times 10^{-6} \text{ m²/s}
\]

SOLUTION

We compare \( K_{\text{be}} \) values for these two cases. From Eq. (19), from experiment,

\[
(K_{\text{be}})_{m=7} = 6 \frac{a_b}{d_b^2} \left( \frac{0.028 - 0.05}{0.13} \right) = 1.29 - 2.31 \text{ s}^{-1}
\]

Now for nonadsorbed gases Eq. (27) gives

\[
K_{\text{be}} = 4.5 \left( \frac{0.0018}{0.13} \right) + 5.85 \left( \frac{9 - 22 \times 10^{-6}}{0.13} \right)^{1/4} = 0.46 - 0.68 \text{ s}^{-1}
\]

Also from Eq. (34)

\[
K_{\text{ce}} = 6.77 \left[ \left( \frac{9 - 22 \times 10^{-6}}{0.13} \right)^{1/2} \right]^{1/2} = 0.27 - 0.43 \text{ s}^{-1}
\]

Combining according to Eq. (14) gives

\[
\frac{1}{(K_{\text{be}})_{m=7}} = \frac{1}{K_{\text{be}}} + \frac{1}{K_{\text{ce}}} = \frac{1}{0.46 - 0.68} + \frac{1}{0.27 - 0.43}
\]

or

\[
(K_{\text{be}})_{m=7} = 0.17 - 0.26
\]

Comparing gives

\[
(K_{\text{be}})_{m=7} = 1.29 - 2.31
\]

Thus, the interchange with adsorbed gas is roughly eight times that of nonadsorbed gas.

PROBLEM

1. Calculate the interchange coefficients based on bubble volume \( K_{\text{be}} \), \( K_{\text{ce}} \), and \( K_{\text{ce}} \) for a helium tracer in a bubbling fluidized bed of nonadsorptive particles. Also determine the overall interchange coefficient based on bed volume \( a_b K_{\text{be}} \).
CHAPTER 11

Particle-to-Gas Mass and Heat Transfer

The previous chapter dealt with the movement of gas in fluidized beds and the interchannel of gas between bubble and emulsion phases. Here we proceed to mass and heat transfer phenomena in fluidized beds, in particular the transfer rates between bed particles and the throughflowing fluidizing gas.

Mass Transfer: Experimental

Single Spheres and Fixed Beds

The mass transfer coefficient $k$ (m/s) for a single sphere of diameter $d_p$ moving through a fluid at relative velocity $u_0$ is given by Froesling [1] as

$$\text{Sh}^* = \frac{k_d u_0 y}{\delta} = 2 + 0.6 \text{Re}^{1/2} \text{Sc}^{1/3}$$  \hspace{1cm} (1)

$$\text{Re}_{ph} = \frac{d_p u_0}{\mu}$$  \hspace{1cm} $\text{Sc} = \frac{\mu}{\rho \delta}$

where $y$ is the logarithmic mean fraction of the inert or noncondensing component, $\delta$ is the gas-phase diffusion coefficient, and Sh, Re, and Sc are the Sherwood, Reynolds, and Schmidt numbers, respectively. For fine particles $u_0$ becomes small, so Sh* is large. In addition, for nonspherical but isometric particles, replace $d_p$ with the screen size $d_s$ and use Eq. (1) as a reasonable approximation. Note that Eq. (1) only applies to single or widely dispersed particles falling through fluids.

For fixed beds of particles of size $d_p$ and sphericity $\phi$, based on the studies of Ranz [2], we may write

$$\text{Sh}^* = 2 + 1.8 \text{Re}_{ph}^{1/2} \text{Sc}^{1/3}$$  \hspace{1cm} (2) \hspace{1cm} \text{for Re}_{ph} > 80