Heat Transfer between Fluidized Beds and Surfaces

One of the remarkable features of the fluidized bed is its temperature uniformity. In practice, this uniformity exists in both the radial and axial directions, even in beds as large as 10 m in diameter. To maintain a given temperature level in the bed requires removing (or adding) a definite amount of heat by contact with an appropriate heat exchange surface. Consequently, a quantitative value of the heat transfer coefficient between surface and bed is needed in design for chemical and physical operations where temperature control is required.

**Experimental Findings**

Heat Transfer Coefficient

The bed-wall heat transfer coefficient $h$ ($W/m^2\cdot K$) is defined by

$$ q = A_w h \Delta T $$

where $q$ is the heat transfer rate ($W$), $A_w$ is the area of the heat exchanger surface, and $\Delta T$ is the mean temperature difference between the bed and surface.

Bed-wall coefficients in gas fluidized beds have been found to be one or two orders of magnitude larger than for gases alone, and since the bed represents a complex interaction of gas and solid, many factors influence the value of $h$.

Numerous experimental studies and correlations for $h$ have been reported in the literature, most limited to a narrow range of conditions. Because of the complex nature of fluidized contacting, these correlations are far from universal. Furthermore, most early experiments were made in small-diameter units for which the flow behavior differs greatly from beds of commercial size.

In connection with catalytic reactors, early reports of $h$ were mainly for

*Let the word wall designate all heat exchange surfaces, whether they be the walls of the vessel or the surface of tubes immersed in the bed.*
CHAPTER 13 — Heat Transfer between Fluidized Beds and Surfaces

beds of fine particles. More recently, fluidized bed combustion of coal has attracted much attention, and this led to numerous studies on coarse particle high-temperature beds. We are thus in a position to correlate \( h \) in a wide range of fluidizing conditions.


**Vertical Tubes and Bed Walls**

Typical of fairly large-sized equipment, Bock and Molerus [5] measured \( h \) for a tube bundle in a \( d_0 = 1 \text{ m} \) bed of fine particles, with results shown in Fig. 1. Their findings show how gas velocity, distributor type, and radial position in the bed affect the measured \( h \) values. Note that \( h = 300-400 \text{ W/m}^2\text{K} \) for \( u_g = 0.2-0.4 \text{ m/s} \), excluding the wall region.

Similar experiments were performed by Selcu et al. [6] with \( d_0 = 0.55 \text{ m} \) and \( d_p = 160 \mu \text{m} \), and by Piepers et al. [7] with \( d_0 = 0.7 \text{ m} \) and \( d_p = 66 \mu \text{m} \). For fine particles and a velocity range of \( u_g = 0.2-0.4 \text{ m/s} \), they reported results similar to those found by Bock and Molerus, or \( h = 200-400 \text{ W/m}^2\text{K} \).

**Horizontal Tubes**

Many experimental studies have been done on heat transfer from fluidized beds to single tubes and tube bundles. Typical of these findings, Fig. 2 displays the data reported by Beeby and Potter [8] in beds of fine solids and shows the effect of particle size, gas velocity, and bed temperature. Note that \( h \) goes through a maximum at some intermediate gas velocity. The decreasing \( h \) at higher \( u_g \) may be attributed to more contact time with bubbles of their very low \( h \) values.

Several groups measured the local but time-averaged \( h \) around the horizontal tubes. At mild fluidizing conditions, namely for \( u_g \) close to \( u_{mf} \), they found low \( h \) values at the bottom and at the top of the tube, and much higher values at the sides of the tubes. This finding can be attributed to voids frequently forming below the tube and by stagnant particles resting on the top of the tube. At higher gas velocities, fluidization becomes more agitated, and this gives a more uniform distribution of \( h \) around the tube. At the usual operating conditions, \( u_g \) is much higher than \( u_{mf} \), hence, it is reasonable to use a mean value for \( h \).

In beds of large and coarse particles and at \( T < 300\text{°C} \) the effect of particle size on \( h \) is reported by Carlonaggio et al. [9] in Fig. 3. A narrow band of \( h \) values well represents their data and the data they collected from other investigators.

As described in Chap. 6, fluidization becomes smoother at high pressure; consequently, \( h \) should increase with pressure. Figure 4, from Bock and Schweinzer [10], shows that this is so. Staub and Canada [11] found similar high-pressure behavior.

In Fig. 5, Glicksman and Decker [12] correlated the results of six groups of investigators, who used \( d_0 = 0.65-4.0 \text{ mm} \) and pressures up to 10 bar. As may be seen, the \( h \) values all fall in a narrow range.
FIGURE 3
Summary of six studies that relate \( h \) to \( d_p \) in coarse particle beds containing horizontal tube bundles: \( L_m = 0.4 - 0.7 \) m, \( d_p = 16 - 32 \) mm, \( \rho \nu = 0.2 - 4 \) m/s; from Caromagno et al. [8].

FIGURE 4
Effect of pressure on \( h \) on horizontal tube bundles in large particle beds, quartz sand \( (d_p = 970 \) \( \mu \)m); adapted from Bock and Schweinzer [10].

FIGURE 5
Correlation for \( h \) in large particle beds at low temperature, \( d_p \) up to 4 mm, pressure up to 10 atm; from Glicksman and Decker [12]. Line A from Eq. (7), line B from Baskakov [13].

Splash Zone
In fluidized combustors containing horizontal tube banks, the total heat transfer rate is often adjusted and controlled not by changing the bed temperature but by raising or lowering the bed level. This changes the fraction of tubes immersed in the bed. How well this procedure works depends on the difference in \( h \) between tubes immersed in the bed and those in the splash zone above the bed surface; hence, one must know the \( h \) values in these zones.

In another application, shallow bed exchangers with horizontal tube banks are being tested as a means for avoiding the fouling of exchanger surfaces with carbonaceous materials, for example, Diesel exhausts.

Interest in these types of applications has led to research on \( h \) in shallow beds and in the splash zone above fluidized beds. Figure 6 shows the range of experimental results of the studies as reported by Kortleven et al. [14] and Grewal et al. [15].

Temperature Effect
Comparing the reported data at high temperature with that at ambient conditions, such as reported in Fig. 3, shows that \( h \) is 100 – 200 W/m²-K higher at 800°C than at ambient conditions. This may be attributed partly to the increase
Experimental Findings

Fluidized beds of fine or intermediate-sized particles. Mickley et al. [16] measured the instantaneous heat transfer coefficient $h_i$ at a point on a vertical 6.35-mm tube located along the axis of a 0.1-m fluidized bed of 43–320 μm particles and found sharply varying $h_i$ values, as shown in Fig. 7. Similar data were also reported by Baskakov et al. [13]. Such findings suggest that the exchanger surface is being bathed alternately by gas bubbles (very low $h_i$ values) and emulsion packets (high $h_i$ values). In addition, this means that $h$ as defined by Eq. (1) only represents a time-averaged value. Mickley et al. [16] also measured the effect of the gas thermal conductivity on the time-averaged $h_{\text{max}}$. Their results are shown in Fig. 8(a).

Martin [17] correlated $h$ with particle size, as shown in Fig. 8(b), using

**FIGURE 8**
Effect of gas thermal conductivity and particle size on $h_{\text{max}}$: (a) from Martin [17]; data from Mickley and Fairbanks [16]; calculated lines from Eq. (19); see Example 2 for line 1; Prob. 2 for line 2; (b) from Martin [17]; dashed line calculated from Eq. (16); see Example 3.

**FIGURE 7**
Instantaneous $h$ on a vertical $d_p = 6.35$-mm heater in a $d_p = 0.1$-m fluidized bed; adapted from Mickley et al. [16].

**FIGURE 6**
$h$ for a horizontal tube bank in and above high-temperature, large particle (~1 mm), fluidized bed coal combustors. Line 0 from Korelev et al. [14], lines 0 and 0 from Grewal et al. [15].

In gas thermal conductivity and partly to the increase in radiant heat transfer at the higher temperatures.

**Objects Immersed in Bubbling Beds**

As a tool for studying the mechanism of heat transfer in fluidized beds, workers have measured $h$ on the surface of single cylinders or spheres immersed in...
data of other workers. Note that $h$ drops sharply for Geldart C solids, that is, when $d_p < 20 \, \mu m$.

**Fast Fluidized and Solid Circulation Systems**

Most of the studies of heat transfer to exchangers in high-velocity systems were done at ambient conditions and with fine particles. We consider some typical results.

Figure 9(a) shows the findings of Guignon et al. [20] in their large experimental unit, Fig. 9(b) displays the results of Furchi et al. [21] in their small-diameter high-velocity unit, and Fig. 10, prepared by Wu et al. [22], correlates $h$ with the density of the suspension flowing past the heat exchange surface. Note that one should also be able to estimate the effect of $G_s/G_v$ on $h$ in the lean phase from Fig. 9(b). These figures all indicate that $h$ decreases as the fraction of suspended solids in the lean-phase mixture is lowered.

**Radiant Heat Transfer**

With radiometer probes, Ozkaynak et al. [27] and Mathur and Saxena [28] directly measured the radiant heat transfer coefficient $h_r$ between hot beds of coarse particles and small vertical surfaces immersed therein. Figure 11(a) shows that the radiation contribution to heat transfer rises rapidly with temperature. Figure 11(b) shows that radiant heat transfer is hindered appreciably when emulsion packets bathe the receiving surface.

Lindsay et al. [29] measured emissivities in the freeboard of a fluidized bed combustor, with the results shown in Fig. 12(a). The overall heat transfer coefficient was also measured. Figure 12(b) displays the theoretical breakdown of the overall heat transfer coefficient and shows that radiation is the major contributor, almost 100 W/m²·K, at this high temperature.

**Figure 9**

$h$ in fast circulating fluidized beds: (a) horizontal tube banks, $d_p = 50 \, \mu m$ in a $1.19 \, m \times 0.79 \, m$ bed of sand; data from Guignon et al. [20]; (b) on the wall of a $d_p = 0.072\, m$ tube of glass beads; data from Furchi et al. [21].

**Figure 10**

Comparison of $h$ in fast fluidized beds with $h$ in ordinary fluidized beds; adapted from Wu et al. [22].

**Figure 11**

Radiant heat transfer from sand beds to vertical surfaces: (a) data from Ozkaynak et al. [27]; (b) adapted from Mathur and Saxena [28].
the measured effective thermal conductivity of fine particles under \( \sigma_{mf} \sim \sigma_{mb} \) conditions.

Florins and Glickman [31] measured \( h \) between a flat surface and a stationary bed of coarse solids as shown in Fig. 13(b), and Botterill [2] and Colakyan and Levenspiel [32] measured \( h \) for moving beds flowing over a horizontal tube.

**Heat Transfer at a Distributor Plate**

Zhang and Ouyang [33] studied this phenomenon with different materials and particle sizes. They noted rapid fluctuations in surface temperature of the distributor and found \( h \) values in the same range as for vessel walls. This suggests that a similar mechanism controls the rate of heat transfer in the two cases.

**Theoretical Studies**

We start by considering heat transfer in fixed and incipiently fluidized beds, and then extend this analysis to bubbling fluidized beds, to the freeboard region, and to fast fluidized beds. Since the thermal characteristics of materials are essential to this development, Table 1 lists these properties for various frequently used gases and solids.

**Fixed and Incipiently Fluidized Beds**

**Fixed Bed with Stagnant Gas.** If heat flows in parallel paths through the gas and the solid, as shown in Fig. 14(a), then the effective thermal conductivity of the fixed bed is given by

\[
k_e = \sigma_{mf} k_f + (1 - \sigma_{mf}) k_s
\]

(2)

However, to account for the actual geometry and the small contact region between adjacent particles as shown in Fig. 14(b), Kinii and Smith [35] developed the following modification to the parallel path model:

\[
k_e = \sigma_{mf} k_f + (1 - \sigma_{mf}) k_s \left( \frac{1}{\phi_s (k_s / k_f) + 2/3} \right)
\]

(3)

**FIGURE 14**

Effective thermal conductivity of a fixed bed: (a) unrealistic parallel path model leads to Eq. (2); (b) more realistic model leads to Eq. (3).
TABLE 1  Thermal Properties of Common Solids and Gases at 20°C

<table>
<thead>
<tr>
<th>Solids</th>
<th>( \rho_s ) (kg/m³)</th>
<th>( C_{ps} ) (J/kg·K)</th>
<th>( k_s ) (W/m·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alumina</td>
<td>4070</td>
<td>910</td>
<td>33</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2707</td>
<td>896</td>
<td>204</td>
</tr>
<tr>
<td>Alundum</td>
<td>3695</td>
<td>778</td>
<td>10</td>
</tr>
<tr>
<td>Brass</td>
<td>8530</td>
<td>490</td>
<td>104</td>
</tr>
<tr>
<td>Carbonotum</td>
<td>3180</td>
<td>640</td>
<td>15</td>
</tr>
<tr>
<td>Coal</td>
<td>1350</td>
<td>1350</td>
<td>0.26</td>
</tr>
<tr>
<td>Coal ash</td>
<td>2400</td>
<td>1350</td>
<td>3.10</td>
</tr>
<tr>
<td>Coke</td>
<td>1350</td>
<td>740</td>
<td>0.90</td>
</tr>
<tr>
<td>Copper</td>
<td>8854</td>
<td>383</td>
<td>388</td>
</tr>
<tr>
<td>Dolomite</td>
<td>3750</td>
<td>880</td>
<td>1.3</td>
</tr>
<tr>
<td>Fire clay</td>
<td>2300</td>
<td>840</td>
<td>0.65</td>
</tr>
<tr>
<td>Glass</td>
<td>2700</td>
<td>765</td>
<td>0.90</td>
</tr>
<tr>
<td>Gravel</td>
<td>2500</td>
<td>840</td>
<td>0.97</td>
</tr>
<tr>
<td>Iron</td>
<td>7897</td>
<td>452</td>
<td>73</td>
</tr>
<tr>
<td>Lead</td>
<td>11,373</td>
<td>130</td>
<td>35</td>
</tr>
<tr>
<td>Lead glass</td>
<td>3000</td>
<td>685</td>
<td>0.93</td>
</tr>
<tr>
<td>Limestone</td>
<td>2500</td>
<td>920</td>
<td>1.7</td>
</tr>
<tr>
<td>Marble</td>
<td>2600</td>
<td>810</td>
<td>2.8</td>
</tr>
<tr>
<td>Polymer</td>
<td>250</td>
<td>1285</td>
<td>0.27</td>
</tr>
<tr>
<td>Quartz</td>
<td>2643</td>
<td>754</td>
<td>0.83</td>
</tr>
<tr>
<td>Sand</td>
<td>2600</td>
<td>840</td>
<td>1.9</td>
</tr>
<tr>
<td>Silica-alumina catalyst</td>
<td>1250</td>
<td>1000</td>
<td>0.56</td>
</tr>
<tr>
<td>SiC</td>
<td>3200</td>
<td>837</td>
<td>18</td>
</tr>
<tr>
<td>Steel</td>
<td>7800</td>
<td>689</td>
<td>45</td>
</tr>
</tbody>
</table>

From Xavier and Davidson [3].

Here, the * refers to stagnant gas conditions, and \( \phi_g = \frac{d_{eq}}{d_p} \) represents the equivalent thickness of gas film around the contact points between particles, which aids in the transport of heat from particle to particle. Since \( \phi_g \) depends on the bed voidage and since we are interested in using Eq. (3) in our fluidized bed development, Fig. 15 gives the values of \( \phi_g \) for the loosest packing of a normal fixed bed, which is at about \( e = 0.476 \).

For most gas-solid systems, \( k_s > k_g \); thus, the last part of the last term in Eq. (3) is smaller than unity. This means that the thermal conductivity of a fixed bed is lower than for the parallel path model of Eq. (2).

**Wall Region with Stagnant Gas.** Consider the wall region to extend a half-particle diameter out from the surface of the heat exchange surface. Then similar to Eq. (3), the thermal conductivity in this layer can be represented by

\[
k_{eq} = \varepsilon_w k_g + \left( 1 - \varepsilon_w \right) k_s \left( \frac{1}{\phi_{eq}(k_s/k_g) + 1/3} \right)
\]

where \( \varepsilon_w \) is the mean void fraction of this wall layer.

Figure 15 also shows the calculated values for \( \phi_{eq} \). Note that the thickness of the equivalent gas layer is greater for particle-wall contact than for particle-particle contact; in addition, because \( \varepsilon_w > \varepsilon_{mf} \), these two factors indicate that the wall layer presents a greater resistance to heat transfer than an equivalent layer in the main body of the bed.

We may now define a heat transfer coefficient for this wall region of

![Figure 15](image-url)

Ratio of effective thickness of gas film around a contact point to particle diameter: \( \phi_{eq} \) for contact between adjacent particles, \( \phi_{w} \) for contact between particle and surface; adapted from Kisti and Smith [38] and Suzuki and Komai [36].
thickness \(d_p/2\), containing stagnant gas, as follows:

\[
h_w^a = \frac{k_w^a}{d_p^2} = \frac{9k_w}{d_p} \quad (5)
\]

**Wall Region with Flowing Gas.** Figure 13(b) shows that heat transfer in fixed beds is enhanced by gas flow through the bed. This can be attributed to the lateral mixing of gas in the void spaces at the surface with adjacent voids. Yagi and Kunii [36] studied this phenomenon and came up with the following two-term expression:

\[
Nu = \frac{h_c d_p}{k_g} = \left( \frac{\text{transfer for no gas flow}}{\text{transfer for gas flow}} \right) + \alpha_w \text{Pr} \text{Re}_p
\]

\[
= \frac{k_w}{d_p} + \alpha_w \text{Pr} \text{Re}_p, \quad \text{Re}_p = \frac{d_p \rho_g \mu}{\mu} < 2000 \quad (6a)
\]

Rearranging this expression gives

\[
h_w = h_w^a + \alpha_w(C_{p_g} \rho_g \mu) = \frac{9k_w}{d_p} + \alpha_w(C_{p_g} \rho_g \mu) \quad (6b)
\]

The lines on Fig. 13(b) are drawn for \(\alpha_w = 0.05\), and the fit to the data shows that this is a reasonable value for \(\alpha_w\) to use in Eq. (6).

**Bubbling Beds—Heat Transfer to Emulsion Packets**

In a bubbling fluidized bed the rising bubbles sweep past the heat exchange surface, thereby washing away the particles resting there and bringing fresh bed particles into direct contact with the surface. Figure 7(b) indicates that the contact time of these packets of emulsion particles with the surface is about 0.2–0.4 s for the conditions of the experiments reported there. More generally this contact time depends on the experimental conditions and the location of the heat exchange surface.

We now consider heat transfer to these packets of particles.

**Large Particles for Short Contact Times.** Here the particles are replaced before their mean temperature can change appreciably, the temperature gradient takes place only within the row of particles in direct contact with the exchange surface, and we can ignore the thermal diffusion into the rest of the emulsion packet. Figure 16(a) shows this situation.

Glicksman and Deckor [12] calculated the "heating" time constant of particles resting on a surface. They found that the temperature of particles larger than 1 mm did not change appreciably for a residence time as long as \(\tau = 1 \text{ s}\). Thus this extreme can be used for these large particles.

Experimental data on these large particle systems, shown in Fig. 5, can be correlated by

\[
h_{\text{packet}} \frac{k_w}{k_g} \frac{1}{(1 - \delta)} = 5.0 + 0.05 \text{Pr} \text{Re}_p \quad (7)
\]

Note the similarity in form with the expression for fixed beds, Eq. (6).

**Small Particles for Long Contact Times.** Here the particles near the surface closely approach the surface temperature; the thermal transient is felt by many layers from the surface, and, hence, thermal diffusion into the emulsion packet becomes the controlling resistance. This extreme is illustrated in Fig. 16(b).

Botterill and Williams [37] solved the unsteady state heat conduction problem for the first layer of particles at the surface and found that \(d_p = 200-\mu m\) particles approach the temperature of the surface in as little as 10 ms. In another estimate of this extreme, Glicksman and Deckor [12] suggested that the temperature of particles contacting a surface changes substantially for particles smaller than 500 \(\mu m\) for a contact time of about 1 s.

To account for thermal diffusion through an emulsion packet, Mickley and Fairbanks [16] proposed the "renewal model." Here the instantaneous heat transfer coefficient at the surface of the packet, after the packet has rested on the surface for time \(t\), is

\[
h_{\text{packet}}(t) = \frac{k_w}{k_g} \frac{1}{(1 - \delta)C_{el}} \left( 1 + \frac{t}{\tau} \right)^{1/2} \quad (8)
\]
Note that this expression ignores the extra resistance right at the exchanger surface where the thermal conductivity and voltage differ from the corresponding values in the main body of the packet.

When all packets of emulsion contact the surface for the same length of time \( t \), the time-averaged heat transfer coefficient becomes

\[
h_{\text{packet}} = \frac{1}{t} \int_0^t h_{\text{packet}} \, dt = 1.13 \left[ \frac{k^* \rho_v \left(1 - \varepsilon_{\text{eff}}\right) C_{p_v} \varepsilon_{\text{wall}}}{1 - \delta_w} \right]^{1/2}
\]

(9)

Since the fraction of time that the surface is bathed by bubbles is equal to the volume fraction of bubbles \( \delta_w \) in the vicinity of the surface, we can show that the bubble frequency \( n_w \) at the surface is related to the emulsion contact time by the simple expression

\[
\tau n_w = 1 - \delta_w
\]

(10)

Combining Eqs. (9) and (10) gives

\[
h_{\text{packet}} = 1.13 \left[ \frac{k^* \rho_v \left(1 - \varepsilon_{\text{eff}}\right) C_{p_v} \varepsilon_{\text{wall}}}{1 - \delta_w} \right]^{1/2}
\]

(11)

**Bubbling Beds — \( h \) at a Heat Exchanger Surface**

We are ready to develop the general expression for the heat transfer coefficient between a fluidized bed and the exchanger surface. This expression should account for the fact that part of the time the surface is bathed by gas and part of the time by emulsion packets:

\[
h = (h_{\text{bubble at surface}} + h_{\text{emulsion at surface}})(1 - \delta_w)
\]

(12)

Now, when a bubble is present at the surface, there are two contributions to heat transfer: radiation and convection. With emissivities of bed solids and wall given by \( \varepsilon_s \) and \( \varepsilon_w \), the radiation coefficient becomes

\[
h_r = \frac{5.67 \times 10^{-8} \left(T_s^4 - T_w^4\right)}{1/\varepsilon_s + 1/\varepsilon_w - 1/\varepsilon_w} \quad \text{[W/m}^2\cdot\text{K]}
\]

(13)

The gas convection contribution when a bubble contacts the surface is normally very small compared to the other contributions to heat transfer. However, for fast fluidized beds and in the freeboard above a dense bed, the fraction of solids is not small, so the convection term can be important. Thus we may write

\[
h_{\text{bubble present}} = h_{\text{radiation}} + h_{\text{gas convection}} = h_r + h_g
\]

(14)

When the emulsion packet is present on the surface, we have heat transfer in series—at the wall region of thickness \( d_p/2 \) followed by transfer at the emulsion packet. In addition, through the wall region we have both convection and radiation. These three terms sum to

\[
h_{\text{emulsion present}} = \frac{1}{1/h_s + 1/h_{\text{wall}} + 1/h_{\text{packet}}} = \frac{1}{1/(h_r + h_w) + 1/h_{\text{packet}}}
\]

(15)

Putting Eq. (6) into Eq. (15), and Eqs. (14) and (15) into Eq. (12) gives the general expression for heat transfer at a surface:

\[
h = \frac{1 - \delta_w}{\left[ \frac{h_r + 2k^* \rho_v d_p}{\alpha_s C_{p_v} \rho_v \varepsilon_{\text{wall}}} + \frac{1}{h_{\text{packet}}} \right] + \frac{1}{h_s}}
\]

(16)

where \( h_{\text{packet}} \) is given by Eq. (11), \( h_r \) by Eq. (13), and \( k^* \) by Eq. (4).

For the Extreme of Fine Particles and High Temperature. Here radiation between emulsion packet and the surface can be ignored because the particles at the surface very quickly approach the surface temperature. Also, gas flow through the emulsion is negligible (small \( \varepsilon_{\text{ball}} \)). Finally, since the wall temperature reaches many particle layers into the emulsion packet resting on the surface, the additional resistance of the first surface layer can be neglected. With these simplifications, Eq. (16) reduces to

\[
h = \delta_w h_r + (1 - \delta_w) h_{\text{packet}}
\]

or

\[
h = \delta_w h_r + 1.13 \left[ k^* \rho_v (1 - \varepsilon_{\text{eff}}) C_{p_v} \varepsilon_{\text{wall}} (1 - \delta_w) \right]^{1/2}
\]

(17)

For the Extreme of Fine Particles and Low Temperature. Here we ignore radiation, so Eq. (17) reduces to

\[
h = 1.13 \left[ k^* \rho_v (1 - \varepsilon_{\text{eff}}) C_{p_v} \varepsilon_{\text{wall}} (1 - \delta_w) \right]^{1/2}
\]

(18)

Line 1 in Fig. 2 shows that this equation can account for the maximum of \( h \) at some intermediate gas velocity. In addition, line 1 in Fig. 8(a) shows that this equation can follow the variations in \( h \) when different gases are used.

For the Extreme of Large Particles. Heat transfer through the emulsion packet can be ignored because the temperature change occurs only in the first layer at the surface. In bubbling beds \( h_g \) can also be ignored. For this situation Eq. (16) reduces to

\[
h = \delta_w h_r + (1 - \delta_w) \left( h_r + \frac{2k^* \rho_v d_p}{\alpha_s} + 0.05 C_{p_v} \rho_v \varepsilon_{\text{wall}} \right)
\]

or

\[
h = h_r + (1 - \delta_w) \left( \frac{2 k^* \rho_v d_p}{\alpha_s} + 0.05 C_{p_v} \rho_v \varepsilon_{\text{wall}} \right)
\]

(19)

Equation (16) can account for the effect of particle size on the heat transfer coefficient; see Fig. 8(b) and Example 3.

**Alternative Theoretical Approaches.** Many models have been proposed to explain the mechanism of heat transfer in fluidized beds [12, 13, 16–18, 34, 38–45]. Some are much too complicated to use for design calculations, and none are general enough to account for all the factors considered in Eq. (16).
CHAPTER 13 — Heat Transfer between Fluidized Beds and Surfaces

h between Moving Beds and Heat Exchange Walls

For gently descending emulsion solids, the residence time of the emulsion in contact with the exchanger wall is very long, the temperature boundary layer extends many particle layers into the bed, no bubbles are present, and radiation can be neglected. In this situation Eq. (16) reduces to

\[ h = \frac{d_p}{3k_{w}^* + 1/h_{\text{packet}}} \]  

(20)

where \( k_{w}^* \) is given by Eq. (4) and \( h_{\text{packet}} \) by Eq. (11).

Freeboard Region, Fast Fluidization, and Circulating Solid Systems

Chapters 7 and 8 mention that a thin layer of fine particles flows along the container walls. Also, when horizontal tubes are present in the freeboard, clusters of particles hit these tubes now and then. This behavior results in fairly high heat transfer rates, as shown in Figs. 6, 9, and 10.

Since the gas velocity is high in these systems, the gas-phase heat transfer coefficient may have to be considered (see discussion above Eq. (15)). Also, the exchanger surfaces are bathed by the lean phase most of the time; thus \( \delta \approx 1 \).

With these conditions, Eq. (16) reduces to

\[ h = h_s + h_g + (1 - \delta_w)h_{\text{packet}} \]  

(21)

and with Eq. (11),

\[ h = h_s + h_g + 1.13[k_{w}^*\rho_s(1 - \varepsilon_{mf})C_{p}p_{w}(1 - \delta_w)]^{1/2} \]  

(22)

Here we assume that the rate at which clumps of emulsion solids hit the tubes is related to the upward flux of solids at that level in the bed, or

\[ (1 - \delta_w)\varepsilon_{mf} \times \text{(upward flux of solids, G_{sw})} \]  

(33)

Recall from Chap. 7 that \( C_{su} \approx \exp(-a\tau) \); combining Eqs. (21)-(33) then gives

\[ \frac{h}{h_s + h_g} \approx \frac{\text{a\tau}^{1/2}}{1 - \delta_w} \]  

(24)

Equation (24) tells us how \( h \) should change with height in the freeboard of a fluidized bed or in a fast fluidized bed.

Values of \( a \) can be estimated from Fig. 7.12 for the freeboard of turbulent beds and from Fig. 9.10 for fast fluidized beds. In Fig. 9 the values of \( h \) calculated with Eq. (24) for fast fluidized beds are compared with experimental data (see Example 4).

The mean heat transfer coefficient in a freeboard-exchanger region of height \( H_f \) is obtained from the expression

\[ h = \frac{1}{H_f} \int_{0}^{H_f} \frac{h \, dz}{H_f} \]  

(25)

EXAMPLE 1

Calculate \( h \) for a horizontal tube bank exchanger immersed in a fluidized bed of fine particles. Compare your results with the experimental data reported in Fig. 2.

Data

<table>
<thead>
<tr>
<th>Solids:</th>
<th>FCC catalyst, ( d_p = 57 \mu m, \rho_p = 940 \text{ kg/m}^3 ), ( C_p = 828 \text{ J/kg-K} ), ( k_s = 0.20 \text{ W/m-K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas:</td>
<td>( k_g = 0.035 \text{ W/m-K} )</td>
</tr>
<tr>
<td>Heat transfer surface:</td>
<td>90 horizontal tubes, 25.4 mm OD, triangular pitch on 50.8-mm centers</td>
</tr>
<tr>
<td>Bed:</td>
<td>( u_{mf} = 6 \text{ mm/s}, \varepsilon_{mf} = 0.476 \text{ (estimate)} )</td>
</tr>
</tbody>
</table>

The bubble frequency at various gas flow rates is estimated as follows:

<table>
<thead>
<tr>
<th>( u_g \text{ (m/s)} )</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{mf} \text{ (in)} )</td>
<td>2</td>
<td>3.1</td>
<td>2.4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

SOLUTION

We give the calculations for \( u_g = 0.2 \text{ m/s} \). Since this is a fine particle bed, we use Eq. (16) to calculate \( h \). But first we evaluate \( \delta \) and \( k_{w}^* \).

Find \( \delta \). To evaluate \( \delta \) we need \( \varepsilon_{mf} \), which should be available from Chap. 6. But for the tube-filled beds of fine particles, no correlations are given, so we extrapolate from related systems and say that bubbles are roughly 1–1.5 times the hydraulic diameter of the tube-filled bed and that their rise velocity is given by Eq. (6.8).

For the hydraulic diameter, Eq. (6.13) gives

\[ d_h = \frac{4(0.0254 \text{ m})(\text{length})}{2(\text{width})} = 0.0508 \text{ m} \]

Hence,

\[ \varepsilon_{mf} = \left( \frac{1 + 1.5}{2} \right)(0.0508) = 0.0635 \text{ m} \]

Then with Eq. (6.7),

\[ u_g = \frac{0.711(0.8 \times 0.0635)^{1/2}}{2} = 0.56 \text{ m/s} \]

Since \( u_g/u_{mf} = (0.56)/(0.476)/(0.006) = 44.5 \), this certainly is a vigorously bubbling bed. Thus from Eq. (6.29),

\[ \delta = \frac{u_{mf}}{u_g} = \frac{0.2}{0.2 - 0.006 + 0.56} = 0.265 \]

Find \( k_{w}^* \). To evaluate \( k_{w}^* \), we first need \( \phi_{b} \). For \( k_s/k_g = 0.20/0.035 = 5.7 \), Fig. 15 gives \( \phi_{b} = 0.19 \). Then Eq. (3) gives

\[ k_{w}^* = 0.476(0.035) + \frac{(1 - 0.476)(0.20)}{(0.19)(5.7) + 2/3} = 0.076 \text{ W/m-K} \]
CHAPTER 13 — Heat Transfer between Fluidized Beds and Surfaces

Inserting all into Eq. (18) and approximating the voidage at the tube walls by the bed voidage, or \( \delta_w = \delta \), gives

\[
h = 1.13[0.0676](640)\left[1 - 0.476(0.829)(3.4)(1 - 0.265)\right]^{1/2} = 315 \text{ W/m\cdotK}
\]

This value is shown as point A in Fig. 2. Similar calculations at other velocities give

\[
\begin{align*}
h &= 271 \text{ W/m}^2 \cdot \text{K} \quad \text{at } u_w = 0.05 \text{ m/s} \\
h &= 290 \text{ W/m}^2 \cdot \text{K} \quad \text{at } u_w = 3.5 \text{ m/s}
\end{align*}
\]

Line 1 on Fig. 2 shows how these equations compare with the reported data.

EXAMPLE 2

Estimate the heat transfer coefficient \( h \) at close to room temperature conditions on a vertical exchanger surface immersed in a small, gently fluidized laboratory-sized fluidized bed of glass beads. Then show how the thermal conductivity of the gas affects the \( h \) value, and compare your results with the reported values displayed in Fig. 8(a).

Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solids:  ( d_p )</td>
<td>80 ( \mu )m</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>2550 kg/m(^3)</td>
</tr>
<tr>
<td>( C_{ps} )</td>
<td>756 J/kg\cdotK</td>
</tr>
<tr>
<td>( k_p )</td>
<td>1.21 W/m\cdotK</td>
</tr>
<tr>
<td>Gas: ( k_g )</td>
<td>0.006, 0.02, and 0.2 W/m\cdotK for different gases</td>
</tr>
</tbody>
</table>

SOLUTION

Start by considering a gas for which \( k_g = 0.02 \) W/m\cdotK. Then since this is a fine particle bed at not too high a temperature, we calculate \( h \) with Eq. (18). To evaluate \( h \) with this equation, we need values for \( \delta, n_w, \) and \( k_g \). Since neither the bubble frequency nor quantities needed to evaluate \( \delta \) (see Example 1) are given, we estimate them. First, for a gently bubbling fluidized bed take

\[ \delta = 0.1-0.3 \quad \text{or, averaging, } \delta = 0.2 \]

From Fig. 5.12, at about 30 cm above the distributor,

\[ n_w = 3 \times 10^{-1} \]

Next \( k_g^* \) for the clump of emulsion resting on the surface is found from Eq. (3). For the voidage of the clump take \( \epsilon_{ert} = 0.476 \) and find \( d_{\phi} \) from Fig. 15. With \( k_g / k_g = 1.21/0.02 = 60.5 \), we get \( d_{\phi} = 0.10 \). Putting these values in Eq. (3) gives

\[
k_g^* = 0.476[0.02 + \left(1 - 0.476\right)(1.21) - 0.104 \text{ W/m\cdotK}]
\]

(0.10)(60.5) + 2/3

Substituting all the values into Eq. (18) gives

\[
h = 1.13[(0.104)(2550)(1 - 0.476)(756)(0.1 - 0.2)]^{1/2} = 567 \text{ W/m}^2 \cdot \text{K}
\]

This value is plotted as point A in Fig. 8(a). Using other values for \( k_g \), say 0.006 and 0.2 W/m\cdotK, allows us to draw line 1.

EXAMPLE 3

Calculate how \( h_{\text{max}} \) changes with particle size in bubbling fluidized beds. Compare the results of these calculations with the experimental findings reported in Fig. 8(b).

Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solids:  ( \rho_p )</td>
<td>2700 kg/m(^3)</td>
</tr>
<tr>
<td>( C_{ps} )</td>
<td>755 J/kg\cdotK</td>
</tr>
<tr>
<td>( k_p )</td>
<td>1.2 W/m\cdotK</td>
</tr>
<tr>
<td>Gas: ( k_g )</td>
<td>0.028 W/m\cdotK</td>
</tr>
</tbody>
</table>

Theoretical Studies

Assume the temperature is not too high, in which case \( h_{\text{max}} \approx 0 \), and ignore transfer when bubble gas contacts the surface, or \( h_{\text{max}} \approx 0 \). To see how the predicted \( h_{\text{max}} \) changes with particle size, start with \( d_p = 10 \text{ mm} \), for which Wunder and Mersmann [19] report \( h_{\text{max}} = 250 \text{ W/m}^2 \cdot \text{K} \), and assume that \( u_g = d_p^{1/2} \). Also assume that the intensity of bubbling is roughly constant, or \( n_w = 5 \) and \( \delta_w = \delta = 0.1 \), for all particle sizes.

SOLUTION

Since we plan to calculate \( h_{\text{max}} \) for the whole range of particle sizes from very small to large, we would be wise to use the general expression for \( h \), Eq. (16). Let us evaluate its various terms.

\[
k_g = 0.028 - 42.9
\]

From Fig. 15 we then get

\[
d_{\phi} = 0.11 \quad \text{and} \quad d_{\phi} = 0.17
\]

From Eq. (3) the effective conductivity of an emulsion packet is

\[
k_g^* = (0.476)(0.028) + \frac{(1 - 0.476)(1.2)}{(0.11)(42.9) + 2/3} = 0.130 \text{ W/m\cdotK}
\]

Substituting into Eq. (11) gives the heat transfer coefficient for the packet of emulsion:

\[
h_{\text{emulsion}} = 0.130 \left[ \frac{0.130 \times (200)(1 - 0.476)(756)(5.0)^{1/2}}{1 - 0.1} \right]^{1/2} = 922.5 \text{ W/m}^2 \cdot \text{K}
\]

From Eq. (4),

\[
k_{\text{max}} = (0.476)(0.028) + \frac{(1 - 0.476)(1.2)}{(0.17)(42.9) + 2/3} = 0.0958 \text{ W/m\cdotK}
\]

Substituting all the above quantities into Eq. (16) gives

\[
h = \frac{1}{2(0.0958) + 0.0958} + \frac{1}{2(0.0958) + 0.0958} = 922.5
\]

Using the known value, \( h = 250 \text{ at } d_p = 10^{-2} \text{ m} \), allows us to evaluate the unknown term in the last equation. This gives

\[
\alpha_{\text{g}} C_{\text{ps}} d_p u_{\phi} = 366.6
\]

We are now ready to calculate \( h \) for other particle sizes. For example, for \( d_p = 2 \text{ mm} = 2 \times 10^{-3} \text{ m} \),

\[
h = \frac{0.9}{2(0.0958)/2(2 \times 10^{-3}) + 366.6(2 \times 10^{-3})^{1/2} + 922.5} = 185 \text{ W/m}^2 \cdot \text{K}
\]

This is shown as point A in Fig. 8(b).

Similar calculations give the dashed curve in Fig. 8(b). Note that Eq. (16) correctly predicts the minimum in \( h_{\text{max}} \) at about \( d_p = 2 \text{ mm} \). On the contrary for very small particles (\( d_p < 20 \mu\text{m} \)), it does not predict the observed sharp fall off in \( h_{\text{max}} \). However, this is where the system is entering the cohesive regime—Geldart C solids—with its very poor fluidization and low \( h \) values.
EXAMPLE 4

Estimate the point heat transfer coefficient at various levels \( z_i \) above the top of the dense bubbling fluidized bed and the mean heat transfer coefficient in the \( H_f = 4 \text{ m} \) high freeboard above a large-diameter, large particle, high-temperature fluidized bed. At the gas velocity to be used, \( u_g = 2.4 \text{ m/s} \), a considerable amount of solid is ejected into the freeboard. Compare your calculated point coefficient with the data reported in Fig. 9(a).

Data. At the bottom of the freeboard region,

\[
h = 350 \text{ W/m}^2\text{K} \quad \text{at} \quad z_i = 0
\]

In the equivalent gas stream, but free of solids,

\[
h = 20 \text{ W/m}^2\text{K}
\]

**SOLUTION**

In a large-diameter fluidized bed, the decay coefficient that characterizes the decrease in density in the freeboard, from Fig. 7.12, is

\[
a = 1.5 \text{ s}^{-1} \quad \text{or} \quad a = \frac{1.5}{2.4} \approx 0.625 \text{ m}^{-1}
\]

With Eq. (24) the point coefficient at height \( z_i \) is

\[
h = \frac{a}{350 - 20} = \frac{1}{0.625}\left(\frac{350 - 20}{2.4}\right) \approx 30 \text{ W/m}^2\text{K}
\]

(1)

This gives

\[
h = 350 \text{ W/m}^2\text{K} \quad \text{at} \quad z_i = 0
\]

and

\[
h = 114 \text{ W/m}^2\text{K} \quad \text{at} \quad z_i = 4 \text{ m}
\]

Equation (1) is plotted in Fig. 9(a) and compares favorably with the reported data there. The mean heat transfer coefficient for the 4-m-high freeboard exchanger is then obtained from Eq. (28). This gives

\[
h = 20 + \frac{2(350 - 20)}{(0.625)^2(4)} \left(1 - \frac{1}{0.625^2 \cdot 2.4}\right) - 20 \text{ W/m}^2\text{K}
\]

**PROBLEMS**

1. Calculate \( h \) on a horizontal tube bundle with wall temperature \( T = 80^\circ\text{C} \) immersed in a 110^\circ\text{C} bed of 82-\( \mu \text{m} \) sand and fluidized by air at various velocities. Compare your results with the data and line 2 of Fig. 2.

Data

- **Solids:** \( \rho_s = 2700 \text{ kg/m}^3, C_{ps} = 756 \text{ J/kg} \cdot ^\circ\text{K}, k_s = 1.2 \text{ W/m} \cdot ^\circ\text{K} \)
- **Air:**
  - at 110^\circ\text{C}, \( k_g = 0.033 \text{ W/m} \cdot ^\circ\text{K} \)
  - at 150^\circ\text{C}, \( k_g = 0.037 \text{ W/m} \cdot ^\circ\text{K} \)
- **Emissivities:** \( \varepsilon_g = 0.8, \varepsilon_s = 0.9, T_w = 80^\circ\text{C} \)

2. Repeat Prob. 1 with the bed at 180^\circ\text{C} and compare your results with the data and line 3 of Fig. 2.

3. A cold heat exchanger tube (120^\circ\text{C}) is immersed in a hot fluidized bed (600^\circ\text{C}) of fine particles. Estimate \( h \) at a point on this tube where the bubble frequency is estimated to be \( n_w = 2 \text{ s}^{-1} \).

Data

- **Solids:** \( \rho_s = 2700 \text{ kg/m}^3, C_{ps} = 756 \text{ J/kg} \cdot ^\circ\text{K}, k_s = 1.2 \text{ W/m} \cdot ^\circ\text{K} \)
- **Gas:** \( k_g = 0.063 \text{ W/m} \cdot ^\circ\text{K}, \)
- **Emissivities:** \( \varepsilon_g = 0.8, \varepsilon_s = 0.9 \)

4. Calculate the radiant heat transfer coefficient at a heat exchanger surface in a bed of coarse particles. Compare your calculation with the data reported in Fig. 11(a).

Data

- **Solids:** \( d_p = 1030 \mu\text{m}, \rho_s = 2700 \text{ kg/m}^3, C_{ps} = 756 \text{ J/kg} \cdot ^\circ\text{K} \)
- **Temperature of bed:** 300^\circ\text{C}, 500^\circ\text{C}, 800^\circ\text{C}
- **Temperature of wall:** 150^\circ\text{C}
- **Emissivities:** \( \varepsilon_s = 0.9, \varepsilon_g = 0.85 \)

5. Estimate \( h \) on a heat exchanger tube located 0.7 m above the surface of a large, dense, vigorously bubbling bed of fine particles.

Data

In the dense bed: \( h = 400 \text{ W/m}^2\text{K} \)

- \( u_g = 0.6 \text{ m/s} \), which is many multiples of \( u_{mf} \)

In the freeboard: \( \alpha = 2 \text{ s}^{-1} \)

Far up in the freeboard where no solids are present:

\[ h_1 + h_2 = 40 \text{ W/m}^2\text{K} \]

**REFERENCES**

7. H. W. Peppers, F. W. Wieczorek, and K. Rietema, in...

---

**References**

- \( u_g \) (m/s) 0.05 0.2 0.4
- \( \delta \) (1) 0.099 0.214 0.38
- \( n_w \) (s\(^{-1}\)) at 110^\circ\text{C} 0.50 1.4 2.4
- \( n_w \) (s\(^{-1}\)) at 180^\circ\text{C} 0.83 1.9 3.0
CHAPTER 13 — Heat Transfer between Fluidized Beds and Surfaces


CHAPTER 14

The RTD and Size Distribution of Solids in Fluidized Beds

In the continuous treatment of solids, fresh particles are fed to a bed and are removed by an overflow pipe or are entrained by the gases. In the bed they may be transformed into a different material (e.g., fresh catalyst into deactivated form, Zn to ZnO), grow (e.g., by deposition from a sprayed evaporating solution, by chemical deposition of silicon from silane gas), or shrink (e.g., by breakup and attrition of friable solid, by sublimation, by combustion of carbon, by chlorination of ilmenite). Individual particles of the same size also have different lengths of stay in the bed. In addition, elutriation, growth, shrinkage, and reaction act differently for the different-sized particles in the bed. All these effects must be accounted for if we wish to control and predict the behavior of a bed that processes solids.

In this chapter we first treat particles of unchanging size. We then treat the more general case of growing and shrinking particles. Except for a few simple cases, numerical methods are needed to find the residence time and size distribution in the bed; however, the procedure is straightforward and follows the same strategy throughout. Our development is limited to steady state operations.

Particles of Unchanging Size

Feed of One Size, Single and Multistage Beds

For a given bed weight and flow rate of solids of unchanging density, as shown in Fig. 1, a material balance gives

$$F_0 = F_1 + F_2 \tag{1}$$

and the mean residence time of solids in a single fluidized bed becomes

$$\bar{\tau} = \frac{W}{F_0} = \frac{W}{F_1 + F_2} \tag{2}$$