Engineer-In-Training Exam File

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ENGINEERING ECONOMICS

We begin this chapter with a brief review of engineering economics. The topics cover the full range of what has appeared on past Engineer-In-Training exams, plus four topics that are likely to appear in the future:

Geometric gradient;

Rate of return analysis of three alternatives;

Benefit–cost ratio;

After-tax economic analysis.

There are 26 example problems scattered throughout the engineering economics review. These examples are an integral part of the review and should be worked to completion as you come to them.

CASH FLOW

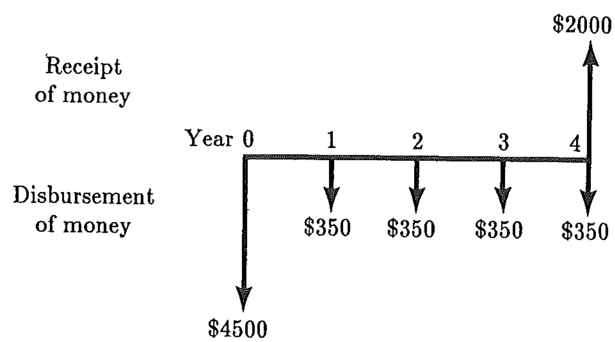
The field of engineering economics uses mathematical and economic techniques to systematically analyze situations which pose alternative courses of action.

The initial step in engineering economics problems is to resolve a situation, or each alternative course in a given situation, into its favorable and unfavorable consequences or factors. These are then measured in some common unit—usually money. Those factors which cannot readily be reduced to money are called intangible, or irreducible, factors. Intangible or irreducible factors are not included in any monetary analysis but are considered in conjunction with such an analysis when making the final decision on proposed courses of action.

A cash flow table shows the "money consequences" of a situation and its timing. For example, a simple problem might be to list the year-by-year consequences of purchasing and owning a used car:

Year	*Cash Flow*	
Beginning of first Year 0	−$4500	Car purchased "now" for $4500 cash. The minus sign indicates a disbursement.
End of Year 1	−350	Maintenance costs are $350 per year.
End of Year 2	−350	
End of Year 3	−350	
End of Year 4	−350	
	+2000	The car is sold at the end of the 4th year for $2000. The plus sign represents a receipt of money.

This same cash flow may be represented graphically:



The upward arrow represents a receipt of money, and the downward arrows represent disbursements. The x-axis represents the passage of time.

EXAMPLE 1

In January 1990, a firm purchases a used typewriter for $500. Repairs cost nothing in 1990 or 1991. Repairs are $85 in 1992, $130 in 1993, and $140 in 1994. The machine is sold in 1994 for $300. Compute the cash flow table.

Solution

Unless otherwise stated in problems, the customary assumption is a beginning-of-year purchase, followed by end-of-year receipts or disbursements, and an end-of-year resale or salvage value. Thus the typewriter repairs and the typewriter sale are assumed to occur at the end of the year. Letting a minus sign represent a disbursement of money, and a plus sign a receipt of money, we are able to set up this cash flow table:

Year	*Cash Flow*
Beginning of 1990	−$500
End of 1990	0
End of 1991	0
End of 1992	−85
End of 1993	−130
End of 1994	+160

Notice that at the end of 1994, the cash flow table shows +160 which is the net sum of −140 and +300. If we define Year 0 as the beginning of 1990, the cash flow table becomes:

Year	*Cash Flow*
0	−$500
1	0
2	0
3	−85
4	−130
5	+160

From this cash flow table, the definitions of Year 0 and Year 1 become clear. Year 0 is defined as the *beginning* of Year 1. Year 1 is the *end* of Year 1. Year 2 is the *end* of Year 2, and so forth.

TIME VALUE OF MONEY

When the money consequences of an alternative occur in a short period of time—say, less than one year—we might simply add up the various sums of money and obtain the net result. But we cannot treat money this same way over longer periods of time. This is because money today is not the same as money at some future time.

Consider this question: Which would you prefer, $100 today or the assurance of receiving $100 a year from now? Clearly, you would prefer the $100 today. If you had the money today, rather than a year from now, you could use it for the year. And if you had no use for it, you could lend it to someone who would pay interest for the privilege of using your money for the year.

EQUIVALENCE

In the preceding section we saw that money at different points in time (for example, $100 today or $100 one year hence) may be equal in the sense that they both are $100, but $100 a year hence is *not* an acceptable substitute for $100 today. When we have acceptable substitutes, we say they are *equivalent* to each other. Thus at 8% interest, $108 a year hence is equivalent to $100 today.

EXAMPLE 2

At a 10% per year interest rate, $500 now is *equivalent* to how much three years hence?

Solution

$500 now will increase by 10% in each of the three years.

Now =			$500
End of 1st year =	500 + 10%(500)	=	550
End of 2nd year =	550 + 10%(550)	=	605
End of 3rd year =	605 + 10%(605)	=	665.50

Thus $500 now is *equivalent* to $665.50 at the end of three years.

Equivalence is an essential factor in engineering economic analysis. Suppose we wish to select the better of two alternatives. First, we must compute their cash flows. An example would be:

	Alternative	
Year	*A*	*B*
0	−$2000	−$2800
1	+800	+1100
2	+800	+1100
3	+800	+1100

The larger investment in Alternative B results in larger subsequent benefits, but we have no direct way of knowing if Alternative B is better than Alternative A. Therefore we do not know which alternative should be selected. To make a decision we must resolve the alternatives into *equivalent* sums so they may be compared accurately and a decision made.

COMPOUND INTEREST FACTORS

To facilitate equivalence computations a series of compound interest factors will be derived and their use illustrated.

Symbols

i = Interest rate per interest period. In equations the interest rate is stated as a decimal (that is, 8% interest is 0.08).

n = Number of interest periods.

P = A present sum of money.

F = A future sum of money. The future sum F is an amount, n interest periods from the present, that is equivalent to P with interest rate i.

A = An end-of-period cash receipt or disbursement in a uniform series continuing for n periods, the entire series equivalent to P or F at interest rate i.

G = Uniform period-by-period increase in cash flows; the arithmetic gradient.

g = Uniform *rate* of period-by-period increase in cash flows; the geometric gradient.

Functional Notation

	To Find	*Given*	*Functional Notation*
Single Payment			
Compound Amount Factor	F	P	$(F/P,i,n)$
Present Worth Factor	P	F	$(P/F,i,n)$

FUNCTIONAL NOTATION, continued

	To Find	Given	Functional Notation
Uniform Payment Series			
Sinking Fund Factor	A	F	$(A/F,i,n)$
Capital Recovery Factor	A	P	$(A/P,i,n)$
Compound Amount Factor	F	A	$(F/A,i,n)$
Present Worth Factor	P	A	$(P/A,i,n)$
Arithmetic Gradient			
Gradient Uniform Series	A	G	$(A/G,i,n)$
Gradient Present Worth	P	G	$(P/G,i,n)$

From the table above we can see that the functional notation scheme is based on writing (To Find / Given,i,n). Thus, if we wished to find the future sum F, given a uniform series of receipts A, the proper compound interest factor to use would be $(F/A,i,n)$.

Single Payment Formulas

Suppose a present sum of money P is invested for one year at interest rate i. At the end of the year, we receive back our initial investment P together with interest equal to Pi or a total amount $P + Pi$. Factoring P, the sum at the end of one year is $P(1 + i)$. If we agree to let our investment remain for subsequent years, the progression is as follows:

	Amount at Beginning of Period	+	Interest for the Period	=	Amount at End of the Period
1st year	P	+	Pi	=	$P(1 + i)$
2nd year	$P(1 + i)$	+	$Pi(1 + i)$	=	$P(1 + i)^2$
3rd year	$P(1 + i)^2$	+	$Pi(1 + i)^2$	=	$P(1 + i)^3$
nth year	$P(1 + i)^{n-1}$	+	$Pi(1 + i)^{n-1}$	=	$P(1 + i)^n$

The present sum P increases in n periods to $P(1 + i)^n$. This gives us a relationship between a present sum P and its equivalent future sum F:

$$\text{Future Sum} = (\text{Present Sum})(1 + i)^n$$
$$F = P(1 + i)^n$$

This is the Single Payment Compound Amount formula. In functional notation it is written:

$$F = P(F/P,i,n)$$

The relationship may be rewritten as:

$$\text{Present Sum} = (\text{Future Sum})(1 + i)^{-n}$$
$$P = F(1 + i)^{-n}$$

This is the Single Payment Present Worth formula. It is written:

$$P = F(P/F,i,n)$$

EXAMPLE 3
At a 10% per year interest rate, $500 now is *equivalent* to how much three years hence?

Solution
This problem was solved in Example 2. Now it can be solved using a single payment formula.

$P =$ \$500
$n =$ 3 years
$i =$ 10%
$F =$ unknown

$$F = P(1 + i)^n = 500(1 + 0.10)^3 = \$665.50$$

This problem may also be solved using the Compound Interest Tables.

$$F = P(F/P,i,n) = 500(F/P,10\%,3)$$

From the 10% Compound Interest Table, read $(F/P,10\%,3) = 1.331$.

$$F = 500(F/P,10\%,3) = 500(1.331) = \$665.50$$

EXAMPLE 4
To raise money for a new business, a man asks you to loan him some money. He offers to pay you $3000 at the end of four years. How much should you give him now if you want 12% interest per year on your money?

Solution

$P =$ unknown
$n =$ 4 years
$i =$ 12%
$F =$ \$3000

$$P = F(1 + i)^{-n} = 3000(1 + 0.12)^{-4} = \$1906.55$$

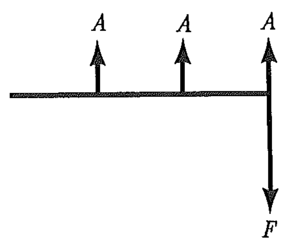
Alternate computation using Compound Interest Tables:

$$P = F(P/F,i,n) = 3000(P/F,12\%,4) = 3000(0.6355) = \$1906.50$$

Note that the solution based on the Compound Interest Table is slightly different from the exact solution using a hand calculator. In economic analysis, the Compound Interest Tables are always considered to be sufficiently accurate.

Uniform Payment Series Formulas

Consider the following situation:



$A =$ End-of-period cash receipt or disbursement in a uniform series continuing for n periods.
$F =$ A future sum of money.

Using the single payment compound amount factor, we can write an equation for F in terms of A:

$$F = A + A(1 + i) + A(1 + i)^2 \qquad (1)$$

In our situation, with $n = 3$, Equation (1) may be written in a more general form:

$$F = A + A(1 + i) + A(1 + i)^{n-1} \qquad (2)$$

Multiply Eq. (2) by $(1 + i)$: $\quad (1 + i)F = A(1 + i) + A(1 + i)^{n-1} + A(1 + i)^n \qquad (3)$

Write Eq. (2): $\quad - \quad F = A + A(1 + i) + A(1 + i)^{n-1} \qquad (2)$

(3) − (2): $\quad iF = -A + A(1 + i)^n$

$$F = A\left(\frac{(1 + i)^n - 1}{i}\right)$$

Uniform Series Compound Amount formula

Solving this equation for A:

$$A = F\left(\frac{i}{(1 + i)^n - 1}\right)$$ Uniform Series Sinking Fund formula

Since $F = P(1 + i)^n$, we can substitute this expression for F in the equation and obtain:

$$A = P\left(\frac{i(1 + i)^n}{(1 + i)^n - 1}\right)$$ Uniform Series Capital Recovery formula

Solving the equation for P:

$$P = A\left(\frac{(1 + i)^n - 1}{i(1 + i)^n}\right)$$ Uniform Series Present Worth formula

In functional notation, the uniform series factors are:

Compound Amount	$(F/A,i,n)$
Sinking Fund	$(A/F,i,n)$
Capital Recovery	$(A/P,i,n)$
Present Worth	$(P/A,i,n)$

EXAMPLE 5

If $100 is deposited at the end of each year in a savings account that pays 6% interest per year, how much will be in the account at the end of five years?

Solution

$$A = \$100$$
$$F = \text{unknown}$$
$$n = 5 \text{ years}$$
$$i = 6\%$$

$$F = A(F/A,i,n) = 100(F/A,6\%,5) = 100(5.637) = \$563.70$$

EXAMPLE 6

A woman wishes to make a uniform deposit every three months to her savings account so that at the end of 10 years she will have $10,000 in the account. If the account earns 6% annual interest, compounded quarterly, how much should she deposit each three months?

Solution

$$F = \$10{,}000$$
$$A = \text{unknown}$$
$$n = 40 \text{ quarterly deposits}$$
$$i = 1\tfrac{1}{2}\% \text{ per quarter year}$$

Note that i, the interest rate per interest period, is $1\frac{1}{2}\%$, and there are 40 deposits.

$$A = F(A/F,i,n) = 10{,}000(A/F,1\tfrac{1}{2}\%,40) = 10{,}000(0.0184) = \$184$$

EXAMPLE 7

An individual is considering the purchase of a used automobile. The total price is $6200 with $1240 as a downpayment and the balance paid in 48 equal monthly payments with interest at 1% per month. The payments are due at the end of each month. Compute the monthly payment.

Solution

The amount to be repaid by the 48 monthly payments is the cost of the automobile *minus* the $1240 downpayment.

$P =$ \$4960
$A =$ unknown
$n =$ 48 monthly payments
$i =$ 1% per month

$$A = P(A/P,i,n) = 4960(A/P,1\%,48) = 4960(0.0263) = \$130.45$$

EXAMPLE 8
A couple sold their home. In addition to cash, they took a mortgage on the house. The mortgage will be paid off by monthly payments of \$232.50 for 10 years. The couple decides to sell the mortgage to a local bank. The bank will buy the mortgage, but requires a 1% per month interest rate on their investment. How much will the bank pay for the mortgage?

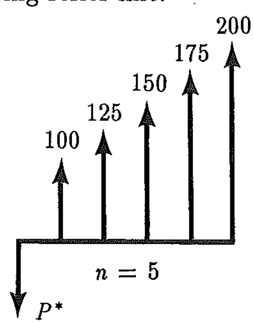
Solution

$A =$ \$232.50
$n =$ 120 months
$i =$ 1% per month
$P =$ unknown

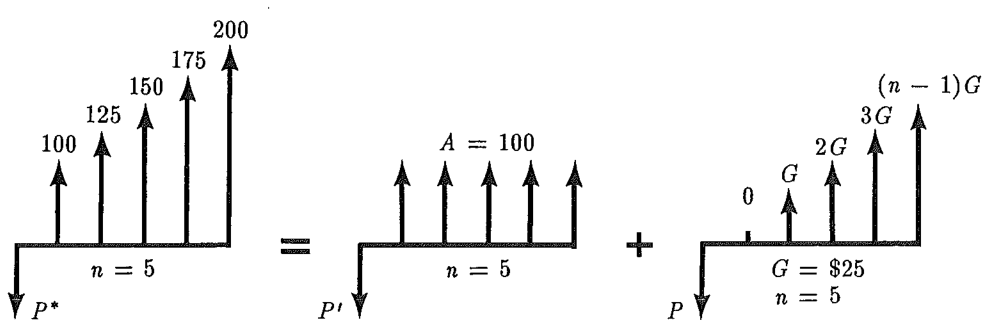
$$P = A(P/A,i,n) = 232.50(P/A,1\%,120) = 232.50(69.701) = \$16,205.48$$

Arithmetic Gradient

At times one will encounter a situation where the cash flow series is not a constant amount A. Instead it is an increasing series like:



This cash flow may be resolved into two components:



We can compute the value of P^* as equal to P' plus P. We already have an equation for P':

$$P' = A(P/A,i,n)$$

The value for P in the right-hand diagram is:

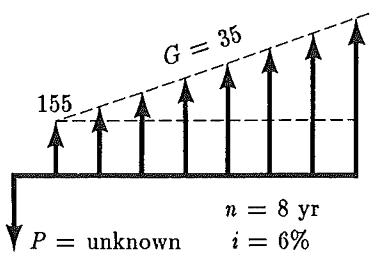
$$P = G\left(\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right)$$

This is the Arithmetic Gradient Present Worth formula. In functional notation, the relationship is $P = G(P/G,i,n)$.

EXAMPLE 9

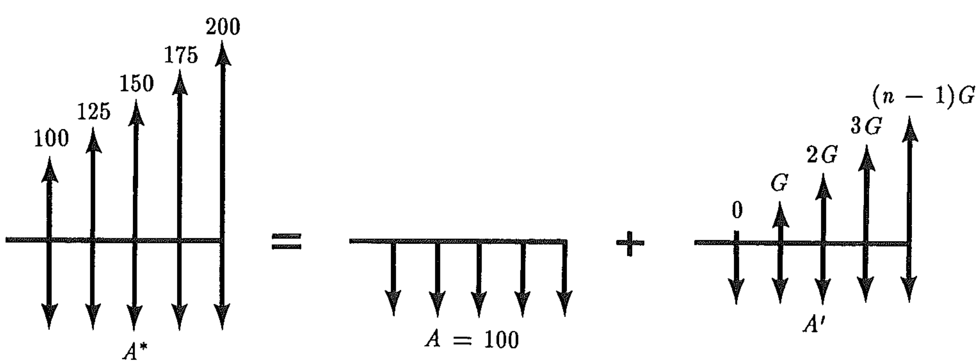
The maintenance on a machine is expected to be $155 at the end of the first year, and increasing $35 each year for the following seven years. What present sum of money would need to be set aside now to pay the maintenance for the eight-year period? Assume 6% interest.

Solution



$$P = 155(P/A,6\%,8) + 35(P/G,6\%,8)$$
$$= 155(6.210) + 35(19.841) = \$1656.99$$

In the gradient series, if instead of the present sum P, an equivalent uniform series A is desired, the problem becomes:



The relationship between A' and G in the right-hand diagram is:

$$A' = G\left(\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right)$$

In functional notation, the Arithmetic Gradient (to) Uniform Series factor is:

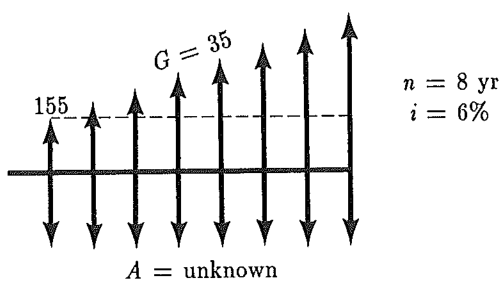
$$A = G(A/G,i,n)$$

It is important to note carefully the diagrams for the two arithmetic gradient series factors. In both cases the first term in the arithmetic gradient series is zero and the last term is $(n - 1)G$. But we use n in the equations and functional notation. The derivations (not shown here) were done on this basis and the arithmetic gradient series Compound Interest Tables are computed this way.

EXAMPLE 10

For the situation in Example 9, we wish now to know the uniform annual maintenance cost. Compute an equivalent A for the maintenance costs.

Solution

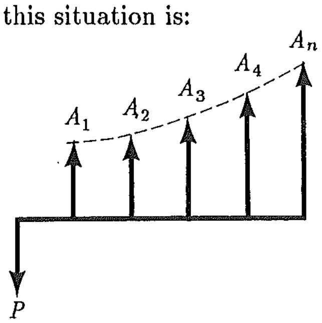


Equivalent uniform annual maintenance cost:

$$\begin{aligned} A &= 155 + 35(A/G,6\%,8) \\ &= 155 + 35(3.195) = \$266.83 \end{aligned}$$

Geometric Gradient

The arithmetic gradient is applicable where the period-by-period change in the cash flow is a uniform amount. There are other situations where the period-by-period change is a *uniform rate*, g. A diagram of this situation is:



where $A_n = A_1(1 + g)^{n-1}$

g = Uniform rate of period-by-period change; the geometric gradient stated as a decimal (8% = 0.08).

A_1 = Value of A at Year 1.

A_n = Value of A at any Year n.

Geometric Series Present Worth Formulas:

When $i = g$, $P = A_1\left(n(1 + i)^{-1}\right)$

When $i \neq g$, $P = A_1\left(\frac{1 - (1 + g)^n(1 + i)^{-n}}{i - g}\right)$

EXAMPLE 11

It is likely that airplane tickets will increase 8% in each of the next four years. The cost of a plane ticket at the end of the first year will be \$180. How much money would need to be placed in a savings account now to have money to pay a student's travel home at the end of each year for the next four years? Assume the savings account pays 5% annual interest.

Solution

The problem describes a geometric gradient where $g = 8\%$ and $i = 5\%$.

$$P = A_1\left(\frac{1 - (1 + g)^n(1 + i)^{-n}}{i - g}\right)$$

$$= 180.00\left(\frac{1 - (1.08)^4(1.05)^{-4}}{0.05 - 0.08}\right) = 180.00\left(\frac{-0.119278}{-0.03}\right) = \$715.67$$

Thus, \$715.67 would need to be deposited now.

As a check, the problem can be solved without using the geometric gradient:

Year		*Ticket*
1	$A_1 =$	\$180.00
2	$A_2 = 180.00 + 8\%(180.00) =$	194.40
3	$A_3 = 194.40 + 8\%(194.40) =$	209.95
4	$A_4 = 209.95 + 8\%(209.95) =$	226.75

$$P = 180.00(P/F,5\%,1) + 194.40(P/F,5\%,2) + 209.95(P/F,5\%,3) + 226.75(P/F,5\%,4)$$

$$= 180.00(0.9524) + 194.40(0.9070) + 209.95(0.8638) + 226.75(0.8227)$$

$$= \$715.66$$

NOMINAL AND EFFECTIVE INTEREST

Nominal interest is the annual interest rate without considering the effect of any compounding.

Effective interest is the annual interest rate taking into account the effect of any compounding during the year.

Frequently an interest rate is described as an annual rate, even though the interest period may be something other than one year. A bank may pay 1–1/2% interest on the amount in a savings account every three months. The *nominal* interest rate in this situation is 6% ($4 \times 1\text{–}1/2\% = 6\%$). But if you deposited \$1000 in such an account, would you have 106%(1000) = \$1060 in the account at the end of one year? The answer is no, you would have more. The amount in the account would increase as follows:

		Amount in Account
	At beginning of year =	\$1000.00
End of 3 months:	$1000.00 + 1\frac{1}{2}\%(1000.00) =$	1015.00
End of 6 months:	$1015.00 + 1\frac{1}{2}\%(1015.00) =$	1030.23
End of 9 months:	$1030.23 + 1\frac{1}{2}\%(1030.23) =$	1045.68
End of one year:	$1045.68 + 1\frac{1}{2}\%(1045.68) =$	1061.37

The actual interest rate on the \$1000 would be the interest, \$61.37, divided by the original \$1000, or 6.137%. We call this the *effective* interest rate.

Effective interest rate $= (1 + i)^m - 1$, where

i = Interest rate per interest period;
m = Number of compoundings per year.

EXAMPLE 12
A bank charges $1\frac{1}{2}\%$ per month on the unpaid balance for purchases made on its credit card. What nominal interest rate is it charging? What effective interest rate?

Solution
The nominal interest rate is simply the annual interest ignoring compounding, or $12(1\frac{1}{2}\%) = 18\%$.

Effective interest rate $= (1 + 0.015)^{12} - 1 = 0.1956 = 19.56\%$

SOLVING ECONOMIC ANALYSIS PROBLEMS

The techniques presented so far illustrate how to convert single amounts of money, and uniform or gradient series of money, into some equivalent sum at another point in time. These compound interest computations are an essential part of economic analysis problems.

The typical situation is that we have a number of alternatives and the question is, which alternative should be selected? The customary method of solution is to resolve each of the alternatives into some common form and then choose the best alternative (taking both the monetary and intangible factors into account).

Criteria

Economic analysis problems inevitably fall into one of three categories:

1. Fixed Input — The amount of money or other input resources is fixed.

   Example: A project engineer has a budget of $450,000 to overhaul a plant.

2. Fixed Output — There is a fixed task, or other output to be accomplished.

   Example: A mechanical contractor has been awarded a fixed price contract to air-condition a building.

3. Neither Input nor Output Fixed — This is the general situation where neither the amount of money or other inputs, nor the amount of benefits or other outputs are fixed.

   Example: A consulting engineering firm has more work available than it can handle. It is considering paying the staff for working evenings to increase the amount of design work it can perform.

There are five major methods of comparing alternatives: present worth; future worth; annual cost; rate of return; and benefit–cost ratio. These are presented in the following sections.

PRESENT WORTH

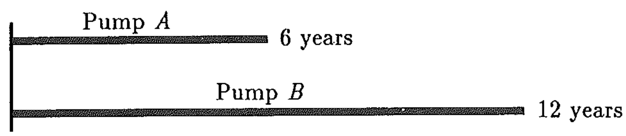
In present worth analysis, the approach is to resolve all the money consequences of an alternative into an equivalent present sum. For the three categories given above, the criteria are:

Category	*Present Worth Criterion*
Fixed Input	Maximize the Present Worth of benefits or other outputs.
Fixed Output	Minimize the Present Worth of costs or other inputs.
Neither Input nor Output Fixed	Maximize [Present Worth of benefits *minus* Present Worth of costs] or, stated another way: Maximize Net Present Worth.

Application of Present Worth

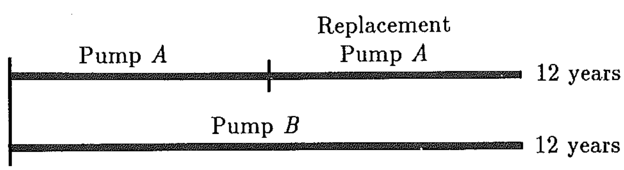
Present worth analysis is most frequently used to determine the present value of future money receipts and disbursements. We might want to know, for example, the present worth of an income producing property, like an oil well. This should provide an estimate of the price at which the property could be bought or sold.

An important restriction in the use of present worth calculations is that there must be a common analysis period when comparing alternatives. It would be incorrect, for example, to compare the present worth (PW) of cost of Pump A, expected to last 6 years, with the PW of cost of Pump B, expected to last 12 years.



Improper Present Worth Comparison

In situations like this, the solution is either to use some other analysis technique* or to restructure the problem so there is a common analysis period. In the example above, a customary assumption would be that a pump is needed for 12 years and that Pump A will be replaced by an identical Pump A at the end of 6 years. This gives a 12-year common analysis period.



Correct Present Worth Comparison

This approach is easy to use when the different lives of the alternatives have a practical least common multiple life. When this is not true (for example, life of J equals 7 years and the life of K equals 11 years), some assumptions must be made to select a suitable common analysis period, or the present worth method should not be used.

EXAMPLE 13

Machine X has an initial cost of \$10,000, annual maintenance of \$500 per year, and no salvage value at the end of its four-year useful life. Machine Y costs \$20,000. The first year there is no maintenance cost. The second year, maintenance is \$100, and increases \$100 per year in subsequent years. The machine has an anticipated \$5000 salvage value at the end of its 12-year useful life.

If interest is 8%, which machine should be selected?

Solution

The analysis period is not stated in the problem. Therefore we select the least common multiple of the lives, or 12 years, as the analysis period.

*Generally the annual cost method is suitable in these situations.

Present Worth of Cost of 12 years of Machine X

$$= 10{,}000 + 10{,}000(P/F,8\%,4) + 10{,}000(P/F,8\%,8) + 500(P/A,8\%,12)$$
$$= 10{,}000 + 10{,}000(0.7350) + 10{,}000(0.5403) + 500(7.536)$$
$$= \$26{,}521$$

Present Worth of Cost of 12 years of Machine Y

$$= 20{,}000 + 100(P/G,8\%,12) - 5000(P/F,8\%,12)$$
$$= 20{,}000 + 100(34.634) - 5000(0.3971)$$
$$= \$21{,}478$$

Choose Machine Y with its smaller PW of Cost.

EXAMPLE 14
Two alternatives have the following cash flows:

	Alternative	
Year	*A*	*B*
0	−$2000	−$2800
1	+800	+1100
2	+800	+1100
3	+800	+1100

At a 5% interest rate, which alternative should be selected?

Solution
Solving by Present Worth analysis:

Net Present Worth (NPW) = PW of benefits − PW of cost

$$\text{NPW}_A = 800(P/A,5\%,3) - 2000$$
$$= 800(2.723) - 2000$$
$$= +178.40$$

$$\text{NPW}_B = 1100(P/A,5\%,3) - 2800$$
$$= 1100(2.723) - 2800$$
$$= +195.30$$

To maximize NPW, choose Alternative B.

Capitalized Cost

In the special situation where the analysis period is infinite ($n = \infty$), an analysis of the present worth of cost is called *capitalized cost.* There are a few public projects where the analysis period is infinity. Other examples would be permanent endowments and cemetery perpetual care.

When n equals infinity, a present sum P will accrue interest of Pi for every future interest period. For the principal sum P to continue undiminished (an essential requirement for n equal to infinity), the end-of-period sum A that can be disbursed is Pi.

$$P \longrightarrow P + Pi \longrightarrow P + Pi \longrightarrow P + Pi \longrightarrow \cdots$$
$$\searrow A \qquad \searrow A \qquad \searrow A$$

When $n = \infty$, the fundamental relationship between P, A, and i is:

$$A = Pi$$

Some form of this equation is used whenever there is a problem with an infinite analysis period.

EXAMPLE 15

In his will, a man wishes to establish a perpetual trust to provide for the maintenance of a small local park. If the annual maintenance is \$7500 per year and the trust account can earn 5% interest, how much money must be set aside in the trust?

Solution

When $n = \infty$, $A = Pi$ or $P = \frac{A}{i}$

$$\text{Capitalized cost } P = \frac{A}{i} = \frac{\$7500}{0.05} = \$150{,}000$$

FUTURE WORTH

In present worth analysis, the comparison is made in terms of the equivalent present costs and benefits. But the analysis need not be made at the present time, it could be made at any point in time: past, present, or future. Although the numerical calculations may look different, the decision is unaffected by the point in time selected. Of course, there are situations where we do want to know what the future situation will be if we take some particular course of action now. When an analysis is made based on some future point in time, it is called future worth analysis.

Category	*Future Worth Criterion*
Fixed Input	Maximize the Future Worth of benefits or other outputs.
Fixed Output	Minimize the Future Worth of costs or other inputs.
Neither Input nor Output Fixed	Maximize [Future Worth of benefits *minus* Future Worth of costs] or, stated another way: Maximize Net Future Worth.

EXAMPLE 16
Two alternatives have the following cash flows:

	Alternative	
Year	*A*	*B*
0	−$2000	−$2800
1	+800	+1100
2	+800	+1100
3	+800	+1100

At a 5% interest rate, which alternative should be selected?

Solution
In Example 14, this problem was solved by Present Worth analysis at Year 0. Here it will be solved by Future Worth analysis at the end of Year 3.

$$\text{Net Future Worth (NFW)} = \text{FW of benefits} - \text{FW of cost}$$

$$\begin{aligned} \text{NFW}_A &= 800(F/A,5\%,3) - 2000(F/P,5\%,3) \\ &= 800(3.152) - 2000(1.158) \\ &= +205.60 \end{aligned}$$

$$\begin{aligned} \text{NFW}_B &= 1100(F/A,5\%,3) - 2800(F/P,5\%,3) \\ &= 1100(3.152) - 2800(1.158) \\ &= +224.80 \end{aligned}$$

To maximize NFW, choose Alternative *B*.

ANNUAL COST

The annual cost method is more accurately described as the method of Equivalent Uniform Annual Cost (EUAC) or, where the computation is of benefits, the method of Equivalent Uniform Annual Benefits (EUAB).

Criteria

For each of the three possible categories of problems, there is an annual cost criterion for economic efficiency.

Category	*Annual Cost Criterion*
Fixed Input	Maximize the Equivalent Uniform Annual Benefits. That is, maximize EUAB.
Fixed Output	Minimize the Equivalent Uniform Annual Cost. That is, minimize EUAC.
Neither Input nor Output Fixed	Maximize [EUAB − EUAC].

Application of Annual Cost Analysis

In the section on present worth, we pointed out that the present worth method requires that there be a common analysis period for all alternatives. This same restriction does not apply in all annual cost calculations, but it is important to understand the circumstances that justify comparing alternatives with different service lives.

Frequently an analysis is to provide for a more or less continuing requirement. One might need to pump water from a well, for example, as a continuing requirement. Regardless of whether the pump has a useful service life of 6 years or 12 years, we would select the one whose annual cost is a minimum. And this would still be the case if the pump useful lives were the more troublesome 7 and 11 years, respectively. Thus, if we can assume a continuing need for an item, an annual cost comparison among alternatives of differing service lives is valid.

The underlying assumption made in these situations is that when the shorter-lived alternative has reached the end of its useful life, it can be replaced with an identical item with identical costs, and so forth. This means the EUAC of the initial alternative is equal to the EUAC for the continuing series of replacements.

If, on the other hand, there is a specific requirement in some situation to pump water for 10 years, then each pump must be evaluated to see what costs will be incurred during the analysis period and what salvage value, if any, may be recovered at the end of the analysis period. The annual cost comparison needs to consider the actual circumstances of the situation.

Examination problems are often readily solved by the annual cost method. And the underlying "continuing requirement" is often present, so that an annual cost comparison of unequal-lived alternatives is an appropriate method of analysis.

EXAMPLE 17

Consider the following alternatives:

	A	*B*
First cost	\$5000	\$10,000
Annual maintenance	500	200
End-of-useful-life salvage value	600	1000
Useful life	5 years	15 years

Based on an 8% interest rate, which alternative should be selected?

Solution

Assuming both alternatives perform the same task and there is a continuing requirement, the goal is to minimize EUAC.

Alternative A:

$$\text{EUAC} = 5000(A/P,8\%,5) + 500 - 600(A/F,8\%,5)$$
$$= 5000(0.2505) + 500 - 600(0.1705) = \$1650$$

Alternative B:

$$\text{EUAC} = 10{,}000(A/P,8\%,15) + 200 - 1000(A/F,8\%,15)$$
$$= 10{,}000(0.1168) + 200 - 1000(0.0368) = \$1331$$

To minimize EUAC, select Alternative *B*.

RATE OF RETURN

A typical situation is a cash flow representing the costs and benefits. The rate of return may be defined as the interest rate where

PW of cost = PW of benefits,

EUAC = EUAB,

or PW of cost − PW of benefits = 0.

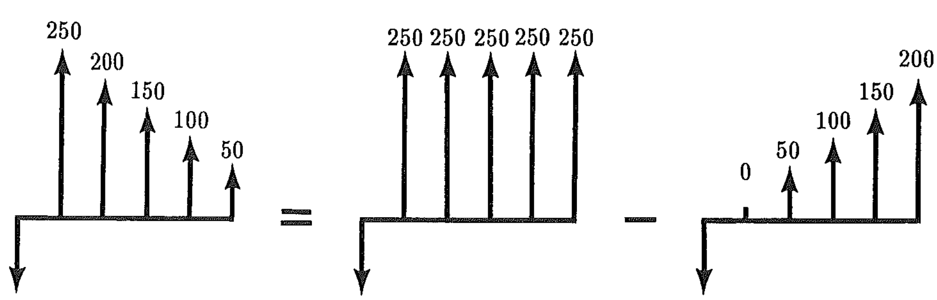
EXAMPLE 18

Compute the rate of return for the investment represented by the following cash flow:

Year	*Cash Flow*
0	−\$595
1	+250
2	+200
3	+150
4	+100
5	+50

Solution

This declining arithmetic gradient series may be separated into two cash flows for which compound interest factors are available:



Note that the gradient series factors are based on an *increasing* gradient. Here, the declining cash flow is solved by subtracting an increasing arithmetic gradient, as indicated by the diagram.

$$\text{PW of cost} - \text{PW of benefits} = 0$$

$$595 - [250(P/A,i,5) - 50(P/G,i,5)] = 0$$

Try $i = 10\%$:

$$595 - [250(3.791) - 50(6.862)] = -9.65$$

Try $i = 12\%$:

$$595 - [250(3.605) - 50(6.397)] = +13.60$$

The rate of return is between 10% and 12%. It may be computed more accurately by linear interpolation:

$$\text{Rate of return} = 10\% + (2\%)\left(\frac{9.65 - 0}{13.60 + 9.65}\right) = 10.83\%$$

Rate of Return Criterion for Two Alternatives

Compute the incremental rate of return on the cash flow representing the difference between the two alternatives. Since we want to look at increments of *investment*, the cash flow for the difference between the alternatives is computed by taking the higher initial-cost alternative *minus* the lower initial-cost alternative. If the incremental rate of return is greater than or equal to the predetermined minimum attractive rate of return (MARR), choose the higher-cost alternative; otherwise, choose the lower-cost alternative.

EXAMPLE 19

Two alternatives have the following cash flows:

	Alternative	
Year	*A*	*B*
0	−\$2000	−\$2800
1	+800	+1100
2	+800	+1100
3	+800	+1100

If 5% is considered the minimum attractive rate of return (MARR), which alternative should be selected?

Solution

These two alternatives were previously examined in Examples 14 and 16 by present worth and future worth analysis. This time, the alternatives will be resolved using rate of return analysis.

Note that the problem statement specifies a 5% minimum attractive rate of raturn (MARR), while Examples 14 and 16 referred to a 5% interest rate. These are really two different ways of saying the same thing: the minimum acceptable time value of money is 5%.

First, tabulate the cash flow that represents the increment of investment between the alternatives. This is done by taking the higher initial-cost alternative minus the lower initial-cost alternative:

Year	Alternative A	Alternative B	Difference between Alternatives B − A
0	−$2000	−$2800	−$800
1	+800	+1100	+300
2	+800	+1100	+300
3	+800	+1100	+300

Then compute the rate of return on the increment of investment represented by the difference between the alternatives:

$$\begin{aligned} \text{PW of cost} &= \text{PW of benefits} \\ 800 &= 300(P/A,i,3) \\ (P/A,i,3) &= \frac{800}{300} = 2.67 \\ i &\approx 6.1\% \end{aligned}$$

Since the incremental rate of return exceeds the 5% MARR, the increment of investment is desirable. Choose the higher-cost Alternative *B*.

Before leaving this example problem, one should note something that relates to the rates of return on Alternative *A* and on Alternative *B*. These rates of return, if computed, are:

	Rate of Return
Alternative *A*	9.7%
Alternative *B*	8.7%

The correct answer to this problem has been shown to be Alternative *B*, and this is true even though Alternative *A* has a higher rate of return. The higher-cost alternative may be thought of as the lower-cost alternative, plus the increment of investment between them. Looked at this way, the higher-cost Alternative *B* is equal to the desirable lower-cost Alternative *A* plus the desirable differences between the alternatives.

The important conclusion is that computing the rate of return for each alternative does not provide the basis for choosing between alternatives. Instead, incremental analysis is required.

EXAMPLE 20

Consider the following:

Year	Alternative A	Alternative B
0	−$200.0	−$131.0
1	+77.6	+48.1
2	+77.6	+48.1
3	+77.6	+48.1

If the minimum attractive rate of return (MARR) is 10%, which alternative should be selected?

Solution

To examine the increment of investment between the alternatives, we will examine the higher initial-cost alternative minus the lower initial-cost alternative, or $A - B$.

	Alternative		*Increment*
Year	*A*	*B*	*A − B*
0	−\$200.0	−\$131.0	−\$69.0
1	+77.6	+48.1	+29.5
2	+77.6	+48.1	+29.5
3	+77.6	+48.1	+29.5

Solve for the incremental rate of return:

$$\text{PW of cost} = \text{PW of benefits}$$
$$69.0 = 29.5(P/A,i,3)$$
$$(P/A,i,3) = \frac{69.0}{29.5} = 2.339$$

From Compound Interest Tables, the incremental rate of return is between 12% and 15%. This is a desirable increment of investment hence we select the higher initial-cost Alternative A.

Rate of Return Criterion for Three or More Alternatives

When there are three or more mutually exclusive alternatives, one must proceed following the same general logic presented for two alternatives. The components of incremental analysis are:

1. Compute the rate of return for each alternative. Reject any alternative where the rate of return is less than the given MARR. (This step is not essential, but helps to immediately identify unacceptable alternatives.)

2. Rank the remaining alternatives in their order of increasing initial cost.

3. Examine the increment of investment between the two lowest-cost alternatives as described for the two-alternative problem. Select the best of the two alternatives and reject the other one.

4. Take the preferred alternative from Step 3. Consider the next higher initial-cost alternative and proceed with another two-alternative comparison.

5. Continue until all alternatives have been examined and the best of the multiple alternatives has been identified.

EXAMPLE 21
Consider the following:

	Alternative	
Year	*A*	*B*
0	−$200.0	−$131.0
1	+77.6	+48.1
2	+77.6	+48.1
3	+77.6	+48.1

If the minimum attractive rate of return (MARR) is 10%, which alternative, if any, should be selected?

Solution
One should carefully note that this is a *three*-alternative problem where the alternatives are A, B, and "Do Nothing."

In this solution we will skip Step 1. Reorganize the problem by placing the alternatives in order of increasing initial cost:

	Do	Alternative	
Year	*Nothing*	*B*	*A*
0	0	−$131.0	−$200.0
1	0	+48.1	+77.6
2	0	+48.1	+77.6
3	0	+48.1	+77.6

Examine the "B − Do Nothing" increment of investment:

Year	B − Do Nothing	
0	−$131.0 − 0 =	−$131.0
1	+48.1 − 0 =	+48.1
2	+48.1 − 0 =	+48.1
3	+48.1 − 0 =	+48.1

Solve for the incremental rate of return:

$$\text{PW of cost} = \text{PW of benefits}$$
$$131.0 = 48.1(P/A,i,3)$$
$$(P/A,i,3) = \frac{131.0}{48.1} = 2.723$$

From Compound Interest Tables, the incremental rate of return = 5%. Since the incremental rate of return is less than 10%, the B − Do Nothing increment is not desirable. Reject Alternative B.

Next, consider the increment of investment between the two remaining alternatives:

Year	A − Do Nothing	
0	−$200.0 − 0 =	−$200.0
1	+77.6 − 0 =	+77.6
2	+77.6 − 0 =	+77.6
3	+77.6 − 0 =	+77.6

Solve for the incremental rate of return:

$$\text{PW of cost} = \text{PW of benefits}$$
$$200.0 = 77.6(P/A,i,3)$$
$$(P/A,i,3) = \frac{200.0}{77.6} = 2.577$$

The incremental rate of return is 8%. Since the rate of return on the A − Do Nothing increment of investment is less than the desired 10%, reject the increment by rejecting Alternative A. We select the remaining alternative: Do nothing!

If you have not already done so, you should go back to Example 20 and see how the slightly changed wording of the problem radically altered it. Example 20 required the choice between two undesirable alternatives. Example 21 adds the Do-nothing alternative which is superior to A or B.

EXAMPLE 22

Consider four mutually exclusive alternatives:

		Alternative		
	A	*B*	*C*	*D*
Initial Cost	$400.0	$100.0	$200.0	$500.0
Uniform Annual Benefit	100.9	27.7	46.2	125.2

Each alternative has a five-year useful life and no salvage value. If the minimum attractive rate of return (MARR) is 6%, which alternative should be selected?

Solution

Mutually exclusive is where selecting one alternative precludes selecting any of the other alternatives. This is the typical "textbook" situation. The solution will follow the several steps in incremental analysis.

1. The rate of return is computed for the four alternatives.

Alternative	*A*	*B*	*C*	*D*
Computed rate of return	8.3%	11.9%	5%	8%

Since Alternative C has a rate of return less than the MARR, it may be eliminated from further consideration.

2. Rank the remaining alternatives in order of increasing intial cost and examine the increment between the two lowest cost alternatives.

Alternative	*B*	*A*	*D*
Initial Cost	$100.0	$400.0	$500.0
Uniform Annual Benefit	27.7	100.9	125.2

	A − B
Δ Initial Cost	$300.0
Δ Uniform Annual Benefit	73.2
Computed Δ rate of return	7%

Since the incremental rate of return exceeds the 6% MARR, the increment of investment is desirable. Alternative A is the better alternative.

3. Take the preferred alternative from the previous step and consider the next higher-cost alternative. Do another two-alternative comparison.

	$D - A$
Δ Initial Cost	\$100.0
Δ Uniform Annual Benefit	24.3
Computed Δ rate of return	6.9%

The incremental rate of return exceeds MARR, hence the increment is desirable. Alternative D is preferred over Alternative A.

Conclusion: Select Alternative D. Note that once again the alternative with the highest rate of return (Alt. B) is *not* the proper choice.

BENEFIT-COST RATIO

Generally in public works and governmental economic analyses, the dominant method of analysis is called benefit–cost ratio. It is simply the ratio of benefits divided by costs, taking into account the time value of money.

$$\text{B/C} = \frac{\text{PW of benefits}}{\text{PW of cost}} = \frac{\text{Equivalent Uniform Annual Benefits}}{\text{Equivalent Uniform Annual Cost}}$$

For a given interest rate, a B/C ratio ≥ 1 reflects an acceptable project. The method of analysis using B/C ratio is parallel to that of rate of return analysis. The same kind of incremental analysis is required.

B/C Ratio Criterion for Two Alternatives

Compute the incremental B/C ratio for the cash flow representing the increment of investment between the higher initial-cost alternative and the lower initial-cost alternative. If this incremental B/C ratio is ≥ 1, choose the higher-cost alternative; otherwise, choose the lower-cost alternative.

B/C Ratio Criterion for Three or More Alternatives

Follow the logic for rate of return, except that the test is whether or not the incremental B/C ratio is ≥ 1.

EXAMPLE 23

Solve Example 22 using Benefit–Cost ratio analysis. Consider four mutually exclusive alternatives:

	Alternative			
	A	*B*	*C*	*D*
Initial Cost	\$400.0	\$100.0	\$200.0	\$500.0
Uniform Annual Benefit	100.9	27.7	46.2	125.2

Each alternative has a five-year useful life and no salvage value. Based on a 6% interest rate, which alternative should be selected?

Solution

1. B/C ratio computed for the alternatives:

$$\text{Alt. } A \qquad \text{B/C} = \frac{\text{PW of benefits}}{\text{PW of cost}} = \frac{100.9(P/A,6\%,5)}{400} = 1.06$$

$$B \qquad \text{B/C} = \frac{27.7(P/A,6\%,5)}{100} = 1.17$$

$$C \qquad \text{B/C} = \frac{46.2(P/A,6\%,5)}{200} = 0.97$$

$$D \qquad \text{B/C} = \frac{125.2(P/A,6\%,5)}{500} = 1.05$$

Alternative *C* with a B/C ratio less than 1 is eliminated.

2. Rank the remaining alternatives in order of increasing intial cost and examine the increment of investment between the two lowest cost alternatives.

Alternative	*B*	*A*	*D*
Initial Cost	\$100.0	\$400.0	\$500.0
Uniform Annual Benefit	27.7	100.9	125.2

	A − *B*
Initial Cost	\$300.0
Uniform Annual Benefit	73.2

$$\text{Incremental B/C ratio} = \frac{73.2(P/A,6\%,5)}{300} = 1.03$$

The incremental B/C ratio exceeds 1.0 hence the increment is desirable. Alternative *A* is preferred over *B*.

3. Do the next two-alternative comparison.

	Alternative		Increment
	A	*D*	*D* − *A*
Initial Cost	\$400.0	\$500.0	\$100.0
Uniform Annual Benefit	100.9	125.2	24.3

$$\text{Incremental B/C ratio} = \frac{24.3(P/A,6\%,5)}{100} = 1.02$$

The incremental B/C ratio exceeds 1.0, hence Alternative D is preferred.

Conclusion: Select Alternative D.

BREAKEVEN ANALYSIS

In business, "breakeven" is defined as the point where income just covers the associated costs. In engineering economics, the breakeven point is more precisely defined as the point where two alternatives are equivalent.

EXAMPLE 24

A city is considering a new $50,000 snowplow. The new machine will operate at a savings of $600 per day, compared to the equipment presently being used. Assume the minimum attractive rate of return (interest rate) is 12% and the machine's life is 10 years with zero resale value at that time. How many days per year must the machine be used to make the investment economical?

Solution

This breakeven problem may be readily solved by annual cost computations. We will set the equivalent uniform annual cost of the snowplow equal to its annual benefit, and solve for the required annual utilization.

Let X = breakeven point = days of operation per year.

$$\text{EUAC} = \text{EUAB}$$

$$50{,}000(A/P{,}12\%{,}10) = 600X$$

$$X = \frac{50{,}000(0.1770)}{600} = 14.7 \text{ days/year}$$

DEPRECIATION

Depreciation of capital equipment is an important component of many after-tax economic analyses. For this reason, one must understand the fundamentals of depreciation accounting.

Depreciation is defined, in its accounting sense, as the systematic allocation of the cost of a capital asset over its useful life. *Book value* is defined as the original cost of an asset, minus the accumulated depreciation of the asset.

In computing a schedule of depreciation charges, three items are considered:

1. Cost of the property, P;
2. Depreciable life in years, n;
3. Salvage value of the property at the end of its depreciable life, S.

Straight Line Depreciation

Depreciation charge in any year $= \frac{P - S}{n}$

Sum-Of-Years-Digits Depreciation

$$\text{Depreciation charge in any year} = \frac{\text{Remaining Depreciable Life at Beginning of Year}}{\text{Sum of Years Digits for Total Useful Life}}(P - S)$$

where Sum Of Years Digits $= 1 + 2 + 3 + \cdots + n = \frac{n}{2}(n + 1)$

Double Declining Balance Depreciation

Depreciation charge in any year $= \frac{2}{n}(P -$ Depreciation charges to date$)$

Accumulated Cost Recovery System (ACRS) Depreciation

ACRS depreciation is based on a property class life which is generally less than the actual useful life of the property and on zero salvage value. The varying depreciation percentage to use must be read from a table (based on declining balance with conversion to straight line). Unless one knows the proper ACRS property class, and has an ACRS depreciation table, the depreciation charge in any year cannot be computed.

EXAMPLE 25

A piece of machinery costs $5000 and has an anticipated $1000 salvage value at the end of its five-year depreciable life. Compute the depreciation schedule for the machinery by:

(a) Straight line depreciation;

(b) Sum-of-years-digits depreciation;

(c) Double declining balance depreciation.

Solution

Straight line depreciation $= \frac{P - S}{n} = \frac{5000 - 1000}{5} = \$800$

Sum-of-years-digits depreciation:

Sum-of-years-digits $= \frac{n}{2}(n + 1) = \frac{5}{2}(6) = 15$

1st year depreciation $= \frac{5}{15}(5000 - 1000) = \quad \$1333$

2nd year depreciation $= \frac{4}{15}(5000 - 1000) =$ 1067

3rd year depreciation $= \frac{3}{15}(5000 - 1000) =$ 800

4th year depreciation $= \frac{2}{15}(5000 - 1000) =$ 533

5th year depreciation $= \frac{1}{15}(5000 - 1000) =$ 267

$4000

Double declining balance depreciation:

1st year depreciation $= \frac{2}{5}(5000 - 0) =$ $2000

2nd year depreciation $= \frac{2}{5}(5000 - 2000) =$ 1200

3rd year depreciation $= \frac{2}{5}(5000 - 3200) =$ 720

4th year depreciation $= \frac{2}{5}(5000 - 3920) =$ 432

5th year depreciation $= \frac{2}{5}(5000 - 4352) =$ 259

$4611

Since the problem specifies a $1000 salvage value, the total depreciation may not exceed $4000. The double declining balance depreciation must be stopped in the 4th year when it totals $4000.

The depreciation schedules computed by the three methods are as follows:

Year	*Straight Line*	*Sum-Of-Years-Digits*	*Double Declining Balance*
1	$800	$1333	$2000
2	800	1067	1200
3	800	800	720
4	800	533	80
5	800	267	0
	$4000	$4000	$4000

INCOME TAXES

Income taxes represent another of the various kinds of disbursements encountered in an economic analysis. The starting point in an after-tax computation is the before-tax cash flow. Generally, the before-tax cash flow contains three types of entries:

1. Disbursements of money to purchase capital assets. These expenditures create no direct tax consequence for they are the exchange of one asset (cash) for another (capital equipment).

2. Periodic receipts and/or disbursements representing operating income and/or expenses. These increase or decrease the year-by-year tax liability of the firm.

3. Receipts of money from the sale of capital assets, usually in the form of a salvage value when the equipment is removed. The tax consequence depends on the relationship between the book value (cost − depreciation taken) of the asset and its salvage value.

Situation	*Tax Consequence*
Salvage value > Book value	Capital gain on difference
Salvage value = Book value	No tax consequence
Salvage value < Book value	Capital loss on difference

After the before-tax cash flow, the next step is to compute the depreciation schedule for any capital assets. Next, taxable income is the taxable component of the before-tax cash flow minus the depreciation. Then, the income tax is the taxable income times the appropriate tax rate. Finally, the after-tax cash flow is the before-tax cash flow adjusted for income taxes.

To organize these data, it is customary to arrange them in the form of a cash flow table, as follows:

Year	*Before-tax cash flow*	*Depreciation*	*Taxable income*	*Income taxes*	*After-tax cash flow*
0	·		·	·	·
1	·	·	·	·	·
·	·	·	·	·	·

EXAMPLE 26

A corporation expects to receive $32,000 each year for 15 years from the sale of a product. There will be an initial investment of $150,000. Manufacturing and sales expenses will be $8067 per year. Assume straight line depreciation, a 15-year useful life and no salvage value. Use a 46% income tax rate.

Determine the projected after-tax rate of return.

Solution

$$\text{Straight line depreciation} = \frac{P - S}{n} = \frac{150{,}000 - 0}{15}$$
$$= \$10{,}000 \text{ per year}$$

Year	*Before-tax cash flow*	*Depreciation*	*Taxable income*	*Income taxes*	*After-tax cash flow*
0	−150,000				−150,000
1	+23,933	10,000	13,933	−6,409	+17,524
2	+23,933	10,000	13,933	−6,409	+17,524
·	·	·	·	·	·
·	·	·	·	·	·
·	·	·	·	·	·
15	+23,933	10,000	13,933	−6,409	+17,524

Take the after-tax cash flow and compute the rate of return at which PW of cost equals PW of benefits.

$$150{,}000 = 17{,}524(P/A,i,15)$$
$$(P/A,i,15) = \frac{150{,}000}{17{,}524} = 8.559$$

From Compound Interest Tables, $i = 8\%$.

A CONCLUDING COMMENT

As you have seen, engineering economics is not a complex subject. There are, however, a group of fundamental concepts and techniques that must be mastered. This brief discussion has sought to provide a sound overview to these fundamentals. Unfortunately, there are likely to be other difficulties that you will encounter. In those situations, you will need to refer to one of the standard textbooks on the subject. I have listed my favorite reference (which should be no surprise) below.

Reference

Newnan, D. G. *Engineering Economic Analysis*, 3rd ed., 1988;
Engineering Press, Inc., P.O. Box 1, San Jose, CA 95103-0001.

ENGR ECON 1

About how long will it take for $10,000 invested at 5% per year, compounded annually, to double in value?

(a) 5 yrs
(b) 10 yrs
(c) 15 yrs
(d) 20 yrs
(e) 25 yrs

P = $10,000 F = $20,000 i = 0.05 n = unknown

Using the single payment compound amount factor

$F = P(1+i)^n$ $1.05^n = \frac{20{,}000}{10{,}000} = 2$ n = 14.2 yrs ●

Alternate solution using compound interest tables

F = P(F/P,5%,n) (F/P,5%,n) = 20,000/10,000 = 2

From 5% interest tables: (F/P,5%,14) = 1.98 (F/P,5%,15) = 2.08

n = 14.2 ●

Answer is (c)